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Travel time and bounded rationality in travellers' route choice behaviour: A computer route choice experiment

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ABSTRACT

Recent empirical studies have found that travellers route choices deviate from perfect rationality, showing that urban trips do not necessary follow the shortest-time routes. However, there is no consent on how much the travellers' route choice behaviour deviates from the perfect rational assumption. The objective of this study is to contribute to the understanding on how travellers process travel time when making route choices, and to quantify to what extent users are strict travel time minimisers or if bounded rationality is observed. The question of whether travellers evaluate travel time differences in absolute or relative terms is also addressed, and the heterogeneity in the route choice behaviour of travellers investigated.

The results of route choice experiments, focused on the choices in diverse OD pairs and traffic conditions, are analysed. In total, 496 participants recorded 5535 choices over 41 OD pairs. It was found that travellers evaluate relative rather than absolute differences in travel time. In 60.5% of the trips participants chose the fastest route, but this percentage is 80% when the travel time between the fastest and the rest of the alternatives is at least 30% higher. Only 10% of the individuals chose the fastest route in all trips, confirming the hypothesis of bounded rationality. Participants exhibited heterogeneous travel time indifference bands: the average participant was indifferent to relative travel time differences of less than 31%. A mixed logit model (MXL), considering heterogeneous indifference bands is proposed. The model shows a similar predictive accuracy compared to the classical MXL model.

1. Introduction

Travel time is often considered the most important variable in explaining the route choice behaviour of travellers (Bovy and Stern, 1990). From an individual point of view, routes with longer travel times result in higher opportunity costs, i.e., less time that the traveller could allocate into other activities (value of time), thus, decreasing the likelihood of being chosen. When studying traffic assignment, it is traditionally assumed that travellers are perfectly rational, in the sense that they know the travel times in all the alternative routes (perfect information) and he or she will always choose the one with the minimum travel time. As a consequence of this hypothesis, the traffic states in a transportation network must fulfil the User Equilibrium (UE) condition, originally stated by Wardrop (1952): "the journey times in all routes actually used are equal and less than those that would be experienced by a single vehicle on any unused route". By relaxing the perfect information assumption (but not the rationality of users), Sheffi (1985) defined the Stochastic User Equilibrium (SUE). At the individual level, SUE means that users are still strict optimisers, i.e., they choose the minimum travel time alternative, but they have no perfect information from the travel times in the system. However, recent empirical studies have shown that travellers do not necessarily choose the minimum travel time route. In the study of Zhu and Levinson (2015), GPS itineraries were collected from 143 residents of the Minneapolis-St. Paul metropolitan area during a period of 13 weeks, finding that 40% of the trips followed the strict shortest-time path. More important, in almost 90% of the trips travellers chose routes no more than 5 min longer than the shortest time route, meaning that users may not be strict optimisers, but consider travel time as a key decision input. In a related study, Yildirimoglu and Kahraman (2018a,b) use GPS trajectories of taxi trips in the city of Shenzhen, China, to compare the actual followed paths to those implied by UE. The results show that 38.2% of the taxi trips followed the shortest-time path. Similar results can also be found in the work of Bekhor et al. (2006), who by analysing data of 188 participants in a survey consisting on the description of their habitual route to work, found that 37% chose a route that overlaps in 90% with the shortest-time alternative, or in the work of Papinski et al. (2009), who examined the GPS traces and survey answers of 31 individuals residing in Ontario,

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Canada. In the survey approximately 50% of the individuals stated that minimising travel is the most important factor in their route choices. These values, however, differ considerably from the results reported in other studies. Hadjidimitriou et al. (2015) analyse the GPS coordinates of 89 travellers in the province of Reggio Emilia, Italy, concluding that only 25% of the trips matched the shortest path route (considering a match to be the routes that overlap at least in 80% with the shortest route). The authors found that travellers selected routes on average 1.3 longer than the shortest path. By analysing the data from an experiment in Virginia, United Estates, involving 20 participants who completed trips on 5 OD pairs over a period of 20 days, Vreeswijk et al. (2014) found that in 74% of the cases the average shortest time route was chosen. However, this percentage varies from 63% to 90% depending on the OD pair. Thomas and Tutert (2010) used license plate observations in the Dutch city of Enschede to conclude that 75% of the trips followed the shortest time paths.

The fact that travellers not necessarily choose the fastest route is explained, on the one hand, by the presence of other route attributes that make some alternatives more desirable than the others. For example, distance, the number of intersections, traffic lights, complexity of the paths, the amount of freeway, aesthetics (Bovy and Stern, 1990; Ramming, 2002; Bekhor et al., 2006; Papinski et al., 2009) and travel time reliability (Avineri and Prashker, 2005; Abdel-Aty et al., 1997; Mahmassani and Liu, 1999). On the other hand, sub-optimal choices of travellers are explained by limitations in travel time perception and cognitive biases that cause deviations from perfect rationality (see Di and Liu (2016) for a review on cognitive biases in route choice behaviour). The cognitive limitations in human reasoning is the cornerstone of bounded rationality of decision makers. Under bounded rationality, decision-makers search a solution until a satisfactory (not necessary optimal) alternative is found, thus departing from perfect rationality. This idea was introduced by Simon (1957) as a model of decision-making process as an alternative to the classical utility maximisation assumption of expected utility or random utility models (Manski and McFadden, 1981; Train, 2003; Walker and Ben-Akiva, 2002). In the context of traffic, bounded rationality was first discussed in Mahmassani and Chang (1987) who introduced the notion of "indifference band" and studied network equilibrium under bounded rationality assumption (BRUE). The idea of indifference band is that travellers are only willing to switch their usual route when time savings are above a threshold. Or, to put it another way, a decision-maker is indifferent to the travel time of the alternatives when their difference is under a threshold (indifference band). The set of alternatives under this condition are called *satisficing*, a term coined by Simon (1957) to refer to alternatives that both satisfy and suffice. By modifying a random utility model to include this threshold, Watling et al. (2018) propose a bounded choice model and formulate the bounded stochastic equilibrium (Bounded SUE). Bounded rationality can therefore explain why travellers do not necessarily choose the shortest-time routes, but close alternatives that may have other appealing features while being considered equivalent from strict travel time point of view.

In the above-cited studies, there is no consent on the amount of travellers that follow the shortest time route, nor the size of the indifference band: the percentage of travellers that chose the fastest route ranges from 25% to 75%. Moreover, five of the six studies are revealed preference (RP), i.e., those based in GPS traces and license plate observations. While RP methods are not affected by validity issues, they have the disadvantage of low control of the experimental environment, meaning that the diversity of explored situations may be limited. In the context of route choice, the list of alternatives and, more important, the related travel times are not known, making it necessary to infer them. This could introduce some errors in the estimates of the proportion of route choices for the shortest time route. Additionally, in some of the experiments the number of participants is small (20 and 31) and thus the estimates may not generalise to the segment of the population under study. The objective of this study is to contribute to the understanding on how travellers process travel time when making route choices, to better quantify to what extent users are strict travel time minimisers, and if bounded rationality is observed, to narrow the estimation of the indifference bands. The question of whether travellers evaluate travel time differences in absolute or relative terms is addressed. Does a difference of 5 min weigh equally in a 10 min trip than in a 30 min trip? The answer to this question is necessary in determining the indifference band. To this purpose, the results of several stated preference route choice computer experiments, using a dedicated simulation game platform, are statistically analysed. Then, a Mixed Logit Model (MXL) (McFadden, 1984; McFadden and Train, 2000; Walker and Ben-Akiva, 2002), estimated considering only satisficing alternatives, is compared to the estimates of an unrestricted MXL model to assess the impact that the indifference bands may have on the route choice probabilities. These models are compared in terms of predictive accuracy for out-of-sample observations.

Computer-based experiments have been largely used to study the route choices of travellers, with particular attention to the study of how travellers learn from experience (Bogers, 2005; Selten et al., 2007), the impact of advanced travel information systems (ATIS) (Adler and McNally, 1994; Mahmassani and Liu, 1999; Ben-Elia and Shiftan, 2010; Abdel-Aty et al., 1997; Srinivasan and Mahmassani, 2000; Bifulco et al., 2014), the effect of travel time variability and risk attitudes on the travellers choices (De Moraes Ramos et al., 2013; Avineri and Prashker, 2005; Bogers et al., 2006), and the impact of human choices on network performance (Iida et al., 1992; Tawfik et al., 2010), to mention some. In this article, experiments focus on travellers' route choices considering travel time information. Participants made choices over 41 OD pairs in the network of the city of Lyon, France, joined by three alternative routes and presented over a map representation of the city. The OD pairs and routes were selected such that the values of their physical attributes (length, directness, number of intersections, number of turns and freeway composition) show a significant variation, while the routes remain being plausible alternatives. Furthermore, the traffic conditions in the network, and thus the travel times in the routes, varied between and within the different experiments. The variability of the route attributes and travel times make it possible to study their joint effect on participants' choices. In total, 496 participants recorded 5535 route choices. From the total number of participants, 71% received travel time estimates in the three alternative routes, eliminating the travel time perception bias from the analysis, and providing a common ground to study the (bounded) rationality in route choice behaviour in the presence of travel time information.

The rest of the article is organised as follows. In Section 2, the route choice experimental tool, the Mobility Decision Game (MDG) is described. The methodology to estimate the perfect and bounded rationality, as well as the size of the indifference band is introduced in Section 3. The specification of a Mixed Logit Model (MXL) that only accounts for satisficing routes is also presented in this section. In Section 4, the results are discussed. First, a global analysis on the travel time minimisation behaviour of travellers is done, including the heterogeneity of the travel time minimisation behaviour by OD pair and by participant. Second, the effect of the absolute (time) and relative (percentage) travel time differences in the travel time minimisation behaviour of travellers is studied. Third, the bounded rationality in the route choice behaviour is analysed, and the heterogeneity of the indifference band is addressed: the distribution of the indifference band is estimated. Finally, the results of the MXL model are analysed to study the effect of the indifference bands on the probability of route choice. This also permits to test the influence of the route attributes on the choices of the participants. A summary of the important results and the main conclusions are presented in Section 5.

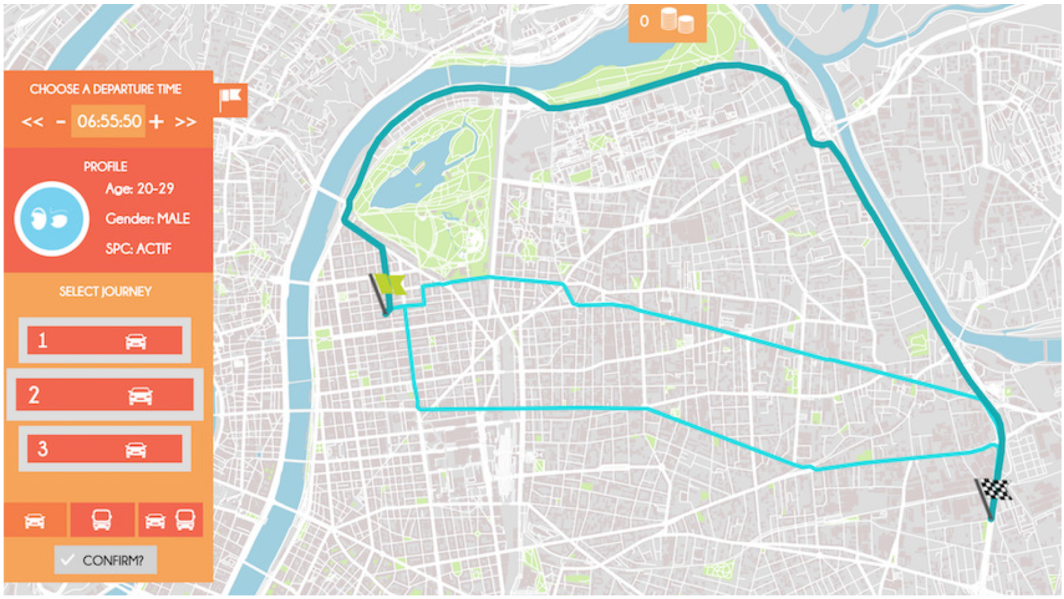

Fig. 1. MDG interface. An OD pair and three alternative routes are shown over a section of the Lyon network in the MDG. The left menu allows participants to choose the route. Note: in the experiments, the mode and departure time buttons in the interface were not activated. Participants could only make route choice decisions.

2. Computer route choice experiments

The computer route choice experiments were carried out using a computer platform, the mobility decision game (MDG), developed to investigate travellers' behaviour in transportation networks at large scale. In an experiment with the MDG, participants are confronted with route choice problems under different scenarios. A route choice problem consist in travelling from an origin to a destination, for which the participants are required to select one of the three proposed routes. A scenario is the environment in which the decision problems are placed: the transportation road network, the OD pairs and routes where the decisions are made, and the traffic conditions. The MDG platform creates the scenarios and present the decision problems to the participants. In the MDG, the participants access simultaneously to the experiment through a dedicated web interface, showing the map of an urban network (see Fig. 1). During a MDG session, multiple OD pairs are assigned to the participants, allowing to observe the choices of the same participants in different OD pairs. Furthermore, some of the participants may receive traffic information in the form of travel time estimates. Thus, the MDG permits to investigate the determinants of the participants' decisions under different transportation and traffic information conditions.

2.1. Scenarios

The road networks in the MDG experiments are based on the real network of the city of Lyon, France. Two networks were used in the experiments: Lyon-36 V, and Lyon-full. The first is composed of 3,663 links and the second of 19,967 links (see Fig. 2). The traffic conditions in the network are dynamically generated by a microscopic traffic simulator, based on the LWR traffic model (Leclercq, 2007; Laval and Leclercq, 2008; Laval and Leclercq, 2010). The simulator generates and handles all the trips that populate the transportation network, based on a trip demand matrix from all origins to all destinations. The MDG runs over a dynamic traffic simulator because it is conceived (i) to reproduce realistic flows in all of the OD configurations during the morning peak hour in Lyon city, giving access to travel times that are close to what can be observed, and (ii) to study the interaction between the participants choices and the traffic states of the network. However, it is important to mention that this last aspect is not relevant to this study, since the travel times that are provided to the participants (which is the main stimulus studied here) are pre-computed and do not depend on the traffic conditions in the experiments. The trips' origins and destinations in the network come from the zoning defined by the National Institute of Statistics and Economic Studies (INSEE) (Institut national de la statistique et des études économiques, 2018), and the major entry/exit points to the network. The most likely routes joining the origins to the destinations are derived with the A^{*} algorithm looking for the k-shortest paths in free-flow travel time. The characteristics of the road networks are summarised in Table 1. The demand scenarios have been built upon the estimation of the real dynamic OD matrix (Krug et al., 2019) with adequate modifications to increase the diversity of travel time configurations for the OD pairs where decision are made. This allows to obtain traffic conditions suitable for testing different behavioural traits. For example, the change in route choice behaviour when the fastest alternative route is switched.

The playable OD pairs and alternative routes are the origins and destinations of the trips that the participants are asked to complete. These OD pairs are predefined in the experiments and are assigned randomly to the participants. Three routes are proposed to the participants to complete a trip between an OD pair in a decision problem. In total, 41 playable OD pairs were defined for the MDG experiments, 15 OD pairs in the Lyon-V36 network and 26 OD pairs in the Lyon-full network. The main idea behind the definition of the OD pairs is the variability of the routes' attributes, both between and within OD pairs. The attributes that are considered in their definition are the euclidean distance from origin to destination, the length of the route, the directness of the trip (euclidean distance divided by the length of the route), the number of turns and crossings per kilometre, and the % of freeway that composes the routes. The values of the attributes of these OD pairs and routes are summarised in Fig. 3, where it can be seen that the distributions of the attributes of the selected OD pairs in the experiments are similar to those of the entire network, i.e., the sample used for the experiments is not that different from a situation that a traveller would find in real life in the Lyon network (González Ramírez et al., 2019). The attributes of the 41 OD pairs and routes are included in Appendix A, and their maps in Appendix B.

In the experiments, the choices of the participants were solely based on the travel time estimates (when available) and the map representation of the routes. The travel time information provided to the participants is computed from the results of a reference simulation, i.e., a previous dynamic simulation of the network, which is parametrised

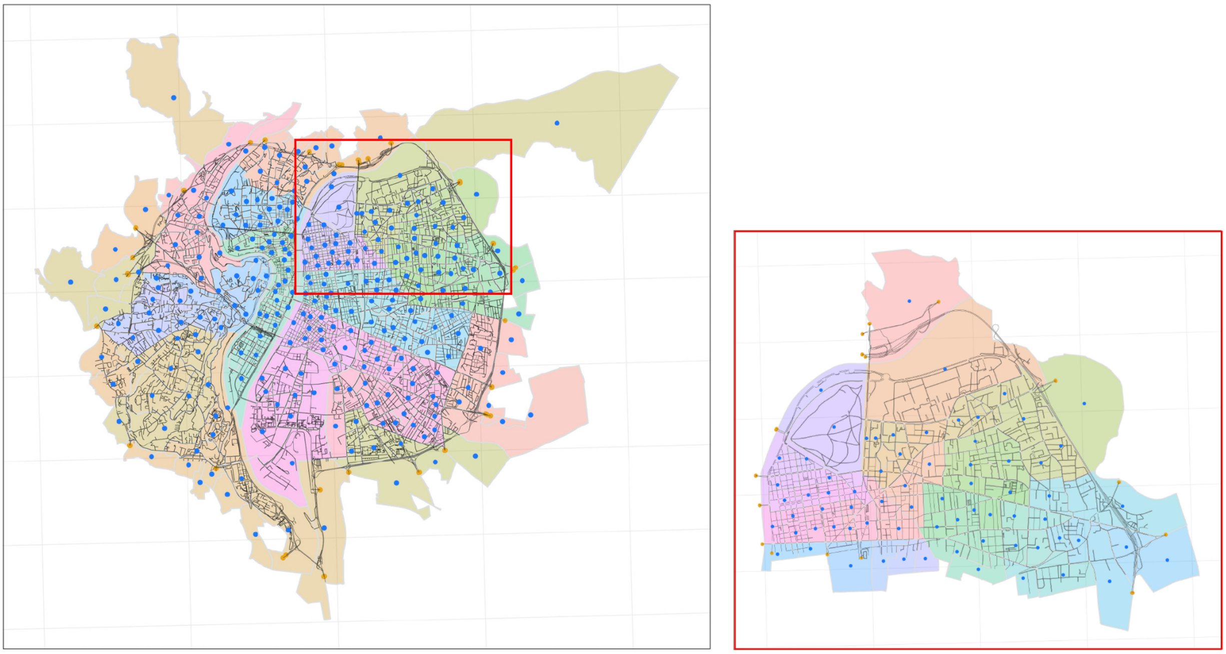

Fig. 2. Lyon-full and Lyon-V36 OD pairs. The zones are depicted in different colours with their centroids in blue. The entry/exit points to the network are depicted with yellow points. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

with the same conditions as in the experiments. The travel time estimates are computed for each link in the network. Since the MDG experiments evolve over a dynamic simulation of the network and during a period of the day (the morning peak), the travel time depends on the fixed departure time of the trip where the route choice is being made.

2.2. Experiment results

The data on route choice behaviour in this article comes from 6 route choice experiments carried out between February 2018 and February 2019. In total, 496 individuals participated in the experiments. The participants in the experiments were students at the University of Lyon taking part in the courses of traffic theory (66%), staff from the IFSTTAR (French Institute of Science and Technology for Transport, Development and Networks) and other universities, who received an invitation by e-mail to remotely join the experiments via a web browser (34%). The great majority of the participants, 80%, are from the city of Lyon, 10% from other cities in France, and 10% from other countries. All participants have signed, before the experiments begin, an informed consent form describing the objectives of the study, the data collection and processing, and the confidentially rules. Participants could opt out of the experiment at any time. No personal data were mandatory to participate to the experiments as people had the opportunity to identify themselves by a login of their choice. Finally, all data were fully anonymised and processed as such. At the beginning of the experiments, the participants were briefed about the objective of the experiment and the interface of the experimental platform; for the participants that joined the experiments via web, a document with the instructions was shared. The participants were instructed to choose the route that they consider the *best* to complete a trip on time.

From the 496 participants, 353 (71%) received traffic information as estimates of the travel time in the alternative routes. The participants recorded a total number of 5535 choices in the 41 OD pairs (Fig. 4). It is important to mention that not all of the 41 OD pairs were available in each experiment in order to guarantee a sufficient number of observations in each one: the maximum number of OD pairs in a single experiment was 15. The distribution of the number of choices per participant is presented in Fig. 5(a), where it can be seen that participants recorded a different number of choices; the average is 11.2. These choices are distributed over an average of 5.41 OD pairs, meaning that participants repeated, on average, 2 choices in the same OD pair (Fig. 5(b)). The variation on the number of choices per participant is explained by the duration of the experiments and the availability of the players: some experiments were carried out in sessions of 30 min while others in sessions of 1 h and participants could opt out of the experiment at any moment.

It is worth mentioning that, even when participants made repeated choices, learning is not observed. The learning process was limited by the design of the experiments, where participants make several simultaneous choices (up to 10), i.e., they do not have to wait until a trip is completed to make the next choice. Furthermore, the OD pairs in the MDG are not presented in any particular order, so participants make choices in other OD pairs before encountering a repetition. As a result, participants might have trouble memorising the travel time information provided in their past choices. This, along with the low number of repetitions in the same OD pair (2 on average) prevented participants from learning. To see this in a quantitative manner, the trend of the percentage of times that the fastest route is chosen is analysed against the ordered choice number. Let $F(t)$ be the percentage of times that the fastest route is chosen in choice number t, then if there is a learning process, one would expect that $F(t+1) - F(t) > 0$, i.e., a positive

Table 1
Characteristics of the Lyon-full and Lyon-V36 road networks.

Network	No. links	No. zones	No. entries	No. exits	No. OD pairs	No. routes	Avg. No. routes OD
Lyon-full	19,967	285	29	28	96,096	559,423	5.8
Lyon-V36	3663	71	14	13	9494	40,938	4.3

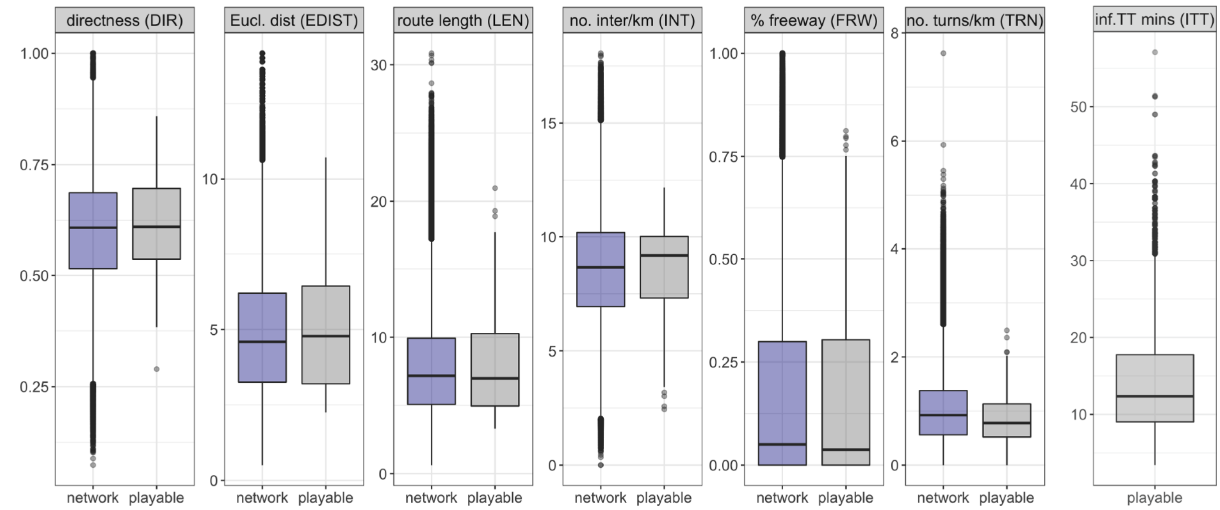

Fig. 3. Distributions of the attributes experienced by the travellers in the OD pairs and three alternative routes defined for the MDG experiments.

trend in the series. The regression $F(t+1) - F(t) = \phi + \epsilon$ is estimated, and the hypothesis test H_0: $\phi \leqslant 0$ is performed. The analysis is done for the participants that received travel time information, as the rest of the article concerns mainly this group. Note that since the number of choices per participant vary (Fig. 5(a)), the values $F(t)$ are obtained with different number of observations: $F(1)$ is estimated with the first choice of participants and thus all participants contribute to its computation; $F(20)$ is estimated with the 20th choice of participants, but there are only around 10% of participants that made at least 20 choices. Therefore the observations need to be weighted in the regression. The result of the regression is $\widehat{phi} = 0.0066$ with a standard error of 0.0136. The test for H_0: $\phi \leqslant 0$ (p-value = 0.3146) suggests that there is not enough evidence to reject the null hypothesis with a high significance level (significance 0.1). Hence, no learning process is suspected. The differences $F(t+1) - F(t)$ are presented in Fig. 6 along with $\hat{\phi}$.

3. Methodology

3.1. Travel time minimisation behaviour

To study to what extent the participants in the experiments are travel time minimisers, the *minimisation rate*, defined as the proportion of times that the fastest route informed to participants was chosen is computed. When travel time is the only variable that travellers take into account when making a route choice, then the minimisation rate can be interpreted as the proportion of perfect rational choices. Denote as $F_{(k)}$

Fig. 4. Choice distributions for the 41 OD pairs in the MDG experiments.

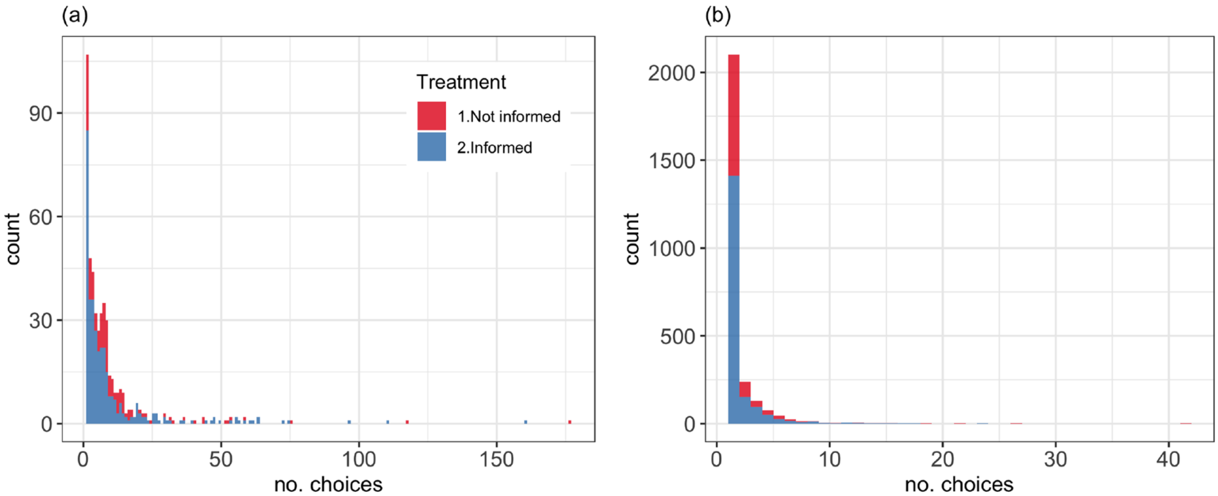

Fig. 5. The distributions of (a) the number of choices per participant and (b) the number of choices per participant per OD pair. This later plot shows that participants barely made more than 2 choices in the same OD pair.

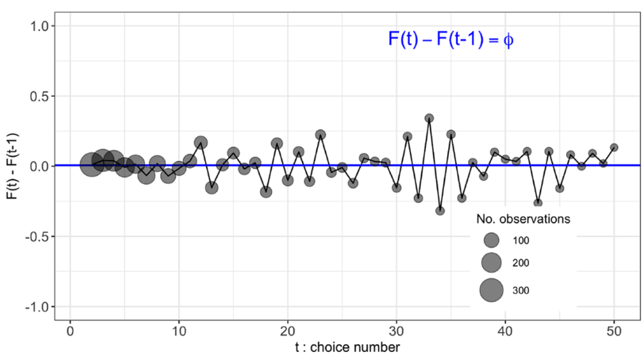

Fig. 6. Differences in the proportion of choices for the fastest route $F(t+1) - F(t)$. If $F(t+1) - F(t) = 0$, then there is no clear trend in the data.

the proportion of times that the kth fastest route informed to participants route was chosen, then $F_{(1)}$ is the minimisation rate. Although the proportions $F_{(k)}$ is defined for the general case, i.e., for choice problems with more than 3 alternative routes, in this study $k = 1, 2, 3$. The proportions $F_{(k)}$, can be computed globally, at OD pair level, at route level, and at participant level. Computed at global level, $F_{(k)}$ allows to make general conclusions about the travel time minimisation behaviour and how this relates to the differences in travel time between the alternative routes. At OD pair and route level, $F_{(k)}$ allows to investigate if the minimisation behaviour is influenced by the characteristics of the routes, other than travel time. Finally, heterogeneity in participants choices can be observed by computing $F_{(k)}$ at participant level. The quantities $F_{(k)}$ can be formally defined in terms of probability. In this article, the terms probability and proportion are used interchangeably.

Let R_{od} be the set of alternative routes belonging to the OD pair od with J alternative routes. Define C as a random variable taking the value $C = j$ $(j = 1, 2, ..., J)$, if the route $r_j \in R_{od}$ is chosen, and $I_{(k)}$ as the random variable taking the value $I_{(k)} = j$ $(j = 1, 2, ..., J)$, when route $r_j \in R_{od}$ is informed to participants to be the kth fastest route. Then, the probability that the route r_j is chosen, given that it was the kth fastest amongst the J alternatives in the OD pair od, is given by

$$F_{(k)}^{j,od} = Pr(C = j | I_{(k)} = j, OD = od). \tag{1}$$

At OD pair level, the proportion of times that the k-th fastest route was chosen, $F_{(k)}^{od}$, is obtained by integrating the expression in Eq. (1) over all the routes $r_j \in R_{od}$, i.e.,

$$\begin{aligned} F_{(k)}^{od} &= \sum_{j|r_j \in R_{od}} Pr\left(C = j \middle| I_{(k)} = j, OD = od\right) \times Pr\left(I_{(k)} = j \middle| OD = od\right) \\ &= \sum_{j|r_j \in R_{od}} Pr\left(C = j, I_{(k)} = j \middle| OD = od\right) = Pr\left(C = I_{(k)} \middle| OD = od\right). \end{aligned} \tag{2}$$

Likewise, the global proportion of times that the kth fastest route was chosen, $F_{(k)}$, is obtained by integrating the expression in Eq. (2) over all OD pairs, this is

$$F_{(k)} = \sum_{od} Pr\left(C = I_{(k)} \middle| OD = od\right) \times Pr\left(OD = od\right) = Pr\left(C = I_{(k)}\right). \tag{3}$$

The proportions $F_{(k)}$ by individual i are obtained in a similar fashion by conditioning by individual instead of the OD pairs, i.e., $F_{(k)}^{i} = Pr(C = I_{(k)}|i)$.

3.2. Travel time boundedly rational behaviour

Recall that under boundedly rational behaviour, a traveller is indifferent to the travel time of the alternatives when their difference is under a threshold (indifference band). The set of alternatives under this condition are called *satisficing*. Analogous to the perfect rational behaviour, the proportion of times that participants chose a satisficing route is computed. Note that the above definition does not mean that the traveller is indifferent to the satisficing routes, in the sense that she or he will choose any of them with the same probability. Rather, the definition means that the effect of travel time is negligible among the satisficing routes. This last interpretation allows for other attributes to play a role in the choices of travellers. Thus, under boundedly rational behaviour, travellers do not necessarily choose the shortest travel time route, but a satisficing route. The indifference band in this study is defined relative to the fastest route.

Let $ITT_{(j)}$ and $ITT_{(k)}$ be the travel time information in the jth and kth fastest routes in a choice problem, such that $ITT_{(j)} \leqslant ITT_{(k)}$. The difference in travel time information can be computed in absolute (time) or relative (percentage) terms as $\Delta ITT_{j,k} = ITT_{(k)} - ITT_{(j)}$, and $\%\Delta ITT_{j,k} = (ITT_{(k)} - ITT_{(j)})/ITT_{(j)}$, respectively. For ease of exposition, in the rest of this section, the differences in the travel time information will be denoted as $\Delta ITT_{j,k}$ to refer to either the absolute or the relative difference. Contrary to the minimisation rate, where each choice problem has a minimum travel time route, in bounded rationality a choice problem may have one, two or more satisficing routes. This implies that for some choice problems the probability of choosing the fastest route needs to be estimated, for other choice problems the probability of choosing the fastest or second fastest, and so on. These probabilities can be written as the conditional probability of choosing a satisficing route, given that there are n satisficing routes. Formally, for a given indifference band $IB(\alpha) = [0, \alpha]$, define the set $S_n(\alpha)$ as the set of choice problems with exactly n satisficing alternative routes. If there are N alternative routes in the choice problems, these sets are

$$\begin{aligned} S_1(\alpha) &= \{choice\,problem \mid \Delta ITT_{1,2} > \alpha\} \\ S_2(\alpha) &= \{choice\,problem \mid \Delta ITT_{1,2} \leqslant \alpha \wedge \Delta ITT_{1,j} > \alpha \quad \forall\ j > 2\} \\ &\dots \\ S_n(\alpha) &= \{choice\,problem \mid \Delta ITT_{1,n} \leqslant \alpha \wedge \Delta ITT_{1,j} > \alpha \quad \forall\ j > n\} \\ &\dots \\ S_N(\alpha) &= \{choice\,problem \mid \Delta ITT_{1,N} \leqslant \alpha\}. \end{aligned} \tag{4}$$

Using the same notation of the previous section, the conditional probabilities of choosing a satisficing route, given that there are n satisficing routes, are then

$$\begin{aligned} Pr\left(satisficing \,\middle|\, S_n(\alpha)\right) &= Pr\left(C \in \bigcup_{k=1}^{n}\{I_{(k)}\} \,\middle|\, S_n(\alpha)\right) \\ &= \sum_{k=1}^{n} Pr\left(C = I_{(k)} \,\middle|\, S_n(\alpha)\right), \end{aligned} \tag{5}$$

where the last equality is because the events $C = I_{(j)}$ and $C = I_{(k)}$ are disjoint for $j \neq k$. Finally, the total probability of choosing a satisficing route can be obtained as

$$Pr\left(satisficing \,\middle|\, \alpha\right) = \sum_{n=1}^{N} Pr\left(satisficing \,\middle|\, S_n(\alpha)\right) \times Pr\left(S_n(\alpha)\right).$$

The probabilities $Pr(satisficing|S_n(\alpha))$ and $Pr(satisficing|\alpha)$ are also estimated for subsamples of the data to create some variation. In total, 141 subsamples are obtained: 41 by removing one OD pair at a time, and 100 by randomly selecting two thirds of the observations with repetition. By sampling two thirds of the observations the margin of error for the estimated proportions is approximately 5%. This sampling strategy (bootstrap) allows to observe the effect that heterogeneous participants and route attributes may have on the estimates of the probabilities.

Note that $Pr(satisficing|S_1(\alpha))$ is *the proportion of times that the fastest route is chosen, given that the difference in the travel time information between the fastest and the rest of the alternatives is more than* α. Since $S_1(\alpha)$ is equivalent to the case when only one route is satisficing (the fastest route), $Pr(satisficing|S_1(\alpha))$ can also be interpreted as the proportion of perfect rational choices. The analysis in Section 4.1.1 is based on the probabilities $Pr(satisficing|S_1(\alpha))$ for different values of α, with special interest in comparing the results between the subsets S_1 when defined in terms of absolute or relative time differences. In Section 4.2, the perfect rationality behaviour is relaxed by considering the cases in which more than one route is satisficing.

3.2.1. Estimation of indifference band by participant

Until now, the bounded rationality has been studied for hypothetical values of α. Moreover, the values of α have been considered to be equal for all the travellers. However, this assumption does not hold (see Section 4.2.1), meaning that travellers are heterogeneous with respect to their indifference bands. To estimate the indifference band of individual i, the travel time differences of the routes chosen by i are considered. Formally, let $C_{i,m}$ represent the chosen route by individual in choice problem m, then by assuming that all the participants choose always a satisficing route, the indifference band of each individual i can be estimated as

$$\hat{\alpha}_i^{max} = max\{\Delta ITT_{1,k} \mid C_{i,m} = k,\ \forall\ m\}.$$

Nevertheless, this definition is restrictive, in the sense that not all information on the travel time differences of the chosen routes is used. For example, a participant that chose a route k with $\Delta ITT_{1,k} = 2$ and the fastest route in the choice problems $m = 2, \dots, M_i$ will have $\hat{\alpha}_i^{max} = 2$, without considering that $\Delta ITT_{1,k'} = 0$ in for all $k' \neq k$. Therefore, two other estimators for α_i are considered in this study: the 95 percentile and the median of the distribution of $\Delta ITT_{1,k} \mid C_{i,m} = k$, with $m = 1, \dots, M_i$. Respectively,

$$Pr(\Delta ITT_{1,k} < \hat{\alpha}_i^{95} \mid C_{i,m} = k,\ \forall\ m) = 0.95$$
$$Pr(\Delta ITT_{1,k} < \hat{\alpha}_i^{50} \mid C_{i,m} = k,\ \forall\ m) = 0.50.$$

3.3. A MXL model for route choice conditioned on the indifference band

The previous sections introduced a methodology to compute the probability of choosing a satisficing route as a function of the indifference band. That methodology allows to make general conclusions about the perfect and bounded rational behaviour of travellers, and assumes no route choice model. Nevertheless, the probability of choosing a specific route is not known. To fill-in this gap, a discrete choice model, specifically, a Mixed Logit Model (MXL) for repeat choices (panel data) is estimated. The model is specified considering the indifference band α_i as an input. α_i is exogenous to the model, and it determines which routes are part of the satisficing set. The routes that do not belong to the satisficing set have probability equal to zero of being chosen.

Let $y_{ij} = 1$ be the event of individual i choosing route j ($y_{ij} = 0$ otherwise) and α_i be its exogenous determined indifference band. For ease of exposition, the subscript for the repeated choices of individuals is eliminated from the notation. Then, the conditional probability of choosing the alternative j can be written as

$$Pr\left(y_{ij} = 1 \,\middle|\, \beta_i, \alpha_i, x_{i\cdot}\right) = \begin{cases} \dfrac{exp\left(x_{ij}^T\beta_i\right)}{\sum_{k\in S(\alpha_i)} exp\left(x_{ik}^T\beta_i\right)}, & \%\Delta ITT_{1,j} \leqslant \alpha_i \\ 0, \quad otherwise, \end{cases} \tag{6}$$

where $x_{i\cdot}$ is the vector of observed attributes of the individual i and routes in the choice problem, β_i is the vector of coefficients specific to the individual, and $S_n(\alpha_i)$ is the set of satisficing routes for the indifference band α_i. The probability in Eq. (6) is conditioned by the vector of coefficients β_i, which represents the preferences or tastes of individual i for the different attributes x_{ij}. MXL models assume that these preferences are drawn from a probability distribution representing the taste heterogeneity between the individuals. Here, it is assumed that $\beta_i \sim \mathcal{N}(\bar{\beta}, \Sigma)$, allowing correlation between preferences. The unconditional choice probability is given by the multiple integral

$$Pr\left(y_{ij} = 1 \,\middle|\, \alpha_i, x_{i\cdot}; \bar{\beta}, \Sigma\right) = \int_{\Omega_\beta} Pr\left(C_i = j \,\middle|\, \beta, \alpha_i, x_{ij}\right) \times Pr\left(\beta \,\middle|\, \bar{\beta}, \Sigma\right) d\beta, \tag{7}$$

where $\bar{\beta}$ and Σ are the parameters defining the distribution of individuals' preferences which need to be estimated.

The above model is a two step process: (1) the individual i conforms the set $S_n(\alpha_i)$ by discarding the not satisficing alternatives from a larger set and, then, (ii) he/she chooses an alternative from $S_n(\alpha_i)$. In the interpretation given here, the first step regards a boundedly rational process, and the second a rational process since the probability in Eq. (6) is given by a MXL model. Note that for a perfect rational traveller $\alpha_i = 0$, which implies that the fastest route is always chosen. It is important to mention that, contrary to random utility models where all the alternatives have a nonzero probability, in Eq. (6) it is possible to assign a zero probability to an alternative in the data that was actually chosen. In other words, if the chosen alternative results to be outside of the indifference band, then it will not be considered in the satisficing set, which violates one of the assumptions of RUMs: the chosen alternative must be part of the chosen set. To avoid this issue, the estimator of the indifference band used as an input is $\hat{\alpha}_i^{max}$, which guarantees that all the chosen routes in the data belong to the satisficing set. The variables that enter the model are:

- FRW_j: the % of freeway that composes the routes;
- DIR_j: the directness of the trip, defined as the euclidean distance divided by the length of the route;
- TRN_j: the number of turns per kilometre;
- INT_j: the number of intersections per kilometre;
- $\%\Delta ITT_{i,(1,j)}$: relative travel time difference between route j and the fastest route; the subindex i indicates that is participant i who received the information.

To estimate the MXL model in this study a Bayesian approach was adopted. In Bayesian inference, a posterior distribution for the parameters $\bar{\beta}$ and Σ is obtained after updating the prior distribution through the likelihood function using the Bayes theorem (see Gelman et al. (2014) for details on Bayesian methods or Train (2001) for Bayesian methods applied to choice modelling). This contrasts with maximum likelihood estimation methods, where point estimates for the parameters are found. The *posterior probability distribution*, denoted as $Pr_{post}(\bar{\beta}, \Sigma)$, has no closed form. However, the Bayesian methods, such as Markov Chain Monte Carlo and Gibbs samplers (Levin and Peres, 2017), allow to obtain samples from this distribution. Estimating the model in Eq. (7) with Bayesian methods has the advantage of providing the means to predict the choices of individuals for which the indifference band has not been observed. To see this, let $\tilde{x}_{i\cdot}$ be the measurable attributes of a new individual and the alternatives in a future choice problem. The *posterior predictive choice probability* for the model in expression (7) is given by

$$\begin{aligned} Pr_{pred}(\tilde{y}_{ij} = 1 | \tilde{x}_{i\cdot}) &= \int Pr(y_{ij} = 1 | \alpha_i, x_{i\cdot}; \bar{\beta}, \Sigma) \times Pr_{post}(\bar{\beta}, \Sigma) d(\bar{\beta}, \Sigma) \\ &\approx \frac{1}{S}\sum_{s=1}^{S} Pr\left(y_{ij} = 1 \middle| \alpha_i, x_{i\cdot}; (\bar{\beta}, \Sigma)_s\right) \\ &= \frac{1}{S}\sum_{s=1}^{S} \int_{\Omega_\beta} Pr\left(y_{ij} = 1 \middle| \beta, \alpha_i, x_{ij}\right) \times Pr\left(\beta \middle| (\bar{\beta}, \Sigma)_s\right) d\beta \\ &\approx \frac{1}{S}\sum_{s=1}^{S} \frac{1}{I}\sum_{i=1}^{I} Pr\left(y_{ij} = 1 \middle| \beta_i^s, \alpha_i, x_{ij}\right), \end{aligned} \quad (8)$$

where $(\bar{\beta}, \Sigma)_s$ is a sample from the posterior $Pr_{post}(\bar{\beta}, \Sigma)$, and β_i^s a sample from $Pr(\beta | (\bar{\beta}, \Sigma)_s)$. The samples β_i^s, for all s, are samples of the posterior distribution of the parameters estimated for the individual i. These samples are readily available after model estimation, the reason is that in Bayesian inference for MXL models the individual coefficients β_i are considered as parameters of the model in order to avoid the integral in expression (7), which may cause numerical instabilities and an increase in computational effort (Train, 2001). Note that in the last equality in expression 8 the indifference bands α_i are treated as individual-specific parameters, and that they are being integrated (along with the coefficients β_i) across individuals. The posterior predictive, $Pr_{pred}(\tilde{y}_{ij} = 1 | \tilde{x}_{i\cdot})$, is therefore the average of the predicted choices of the individuals in the training set, where each individual has its own indifference band.

The *log pointwise predictive density* (*lppd*), a measure of goodness-of-fit of a model, is the Bayesian analogous of the log-likelihood, and it is obtained by considering the joint posterior predictive probability. Let $\mathscr{D}$ be a set of observations (they can be future or observed choices). Then, assuming independent observations, the *lppd* is given by

$$\begin{aligned} lppd = logPr^{pred}(\mathscr{D}) &= log \prod_i \prod_k [Pr_{pred}(y_{ik} = 1 | x_{i\cdot})]^{y_{ik}} \\ &= \sum_i \sum_k [y_{ik} \times logPr_{pred}(y_{ik} = 1 | x_{i\cdot})], \end{aligned} \quad (9)$$

where $(y_{ik}, x_{i\cdot})$ are observed and they can be in or out-of-sample. When the elements in $\mathscr{D}$ are the same used to fit the model, then *lppd* is a measure of goodness-of-fit. When the elements in $\mathscr{D}$ are out-of-sample observations, then *lppd* is a measure of predictive error. Although the *lppd* does not give an absolute scale to evaluate a model, it can be compared between different models, and therefore used for model selection. Higher values of *lppd* are desirable. Observe that only the actual chosen alternatives ($y_{ik} = 1$) contribute in the computation of the *lppd*. In some applications, however, it is of interest to evaluate the aggregated predicted probability for each alternative, i.e.,

$$\bar{Pr}_{pred}(j) = \frac{1}{I}\sum_{i=1}^{I} Pr_{pred}\left(y_{ij} = 1 \middle| x_{i\cdot}\right). \quad (10)$$

This is the case in route choice, in which the aggregated choice probability represents the predicted usage of the routes. Therefore, second way to test the model's performance is to measure the discrepancy between the observed and predicted choice distributions. The following measure is proposed to measure this discrepancy

$$err\left(Pr_{obs}, \bar{Pr}_{pred}\right) = \sum_{j=1}^{J} max\left(0, Pr_{obs}(j) - \bar{Pr}_{pred}(j)\right), \quad (11)$$

where J is the number of alternatives. Fig. 7 shows an example of how *err* is computed. An advantage of this definition is that the error is in probability units, for example, $err = 0.1$ means that 10% of the choices will be erroneous on average. Note that in this article there are 41 OD pairs, therefore *err* needs to be computed separately for each OD pair and then averaged to obtain the global error.

4. Results

4.1. Perfect rationality

The distribution of the choices of the participants amongst the fastest, second fastest and slow routes is presented in Table 2. In the results of the route choice experiments, there is clear difference between the choices of participants who received travel time information (informed participants) and those who did not. The difference between the informed participants and those who did not receive information is confirmed by a χ^2 test, rejecting the null hypothesis (with a significance level of $\alpha = 0.001$) that the observed distributions are the same. Therefore, it can be concluded that information has an impact on the choices of the participants, and that the most preferred routes (in the case of no information) differ from those with information, meaning that travel time information is not necessarily aligned to the preferred route' attributes. The most notable difference between these

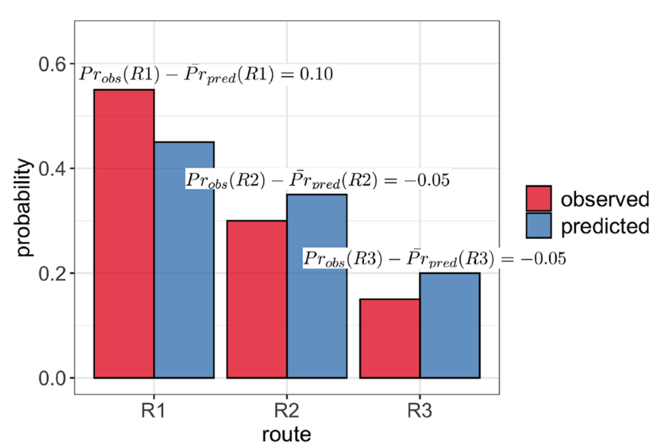

Fig. 7. Example of the computation of *err* for a given OD pair. In this case, $err(Pr_{obs}, \bar{Pr}_{pred}) = 0.10$.

Table 2
Global percentage of times that the fastest, second fastest and third fastest routes were chosen.

Travel time info.	$F_{(1)}$	$F_{(2)}$	$F_{(3)}$
No	0.460	0.328	0.212
Yes	0.605	0.236	0.159

$\chi^2 = 106$, df $= 2$, p-value $< 2.2e-16$

distributions is for $F_{(1)}$, thus, it can be concluded that travel time information causes a minimisation behaviour in participants. It is interesting to note that $F_{(1)} > F_{(2)} > F_{(3)}$ in the not informed case, suggesting that participants are somehow minimising the travel time by choosing the routes with the characteristics that they believe lead to smaller travel times. A second observation is that informed participants preferred slower routes in nearly 40% of the cases, meaning that they are not necessarily strict travel time minimisers. Could this behaviour be explained by bounded rationality? Are there factors other than travel time influencing the choices of the participants? This question will be further investigated.

The minimisation rate in each of the OD pairs, $F_{(1)}^{od}$, is presented in Fig. 8(a), where it can be seen that $F_{(1)}^{od}$ shows a high variability, with values of $F_{(1)}^{od}$ ranging between 0.27 and 0.92. In the case of the three OD pairs with the largest minimisation rates, the alternative with the high composition of freeway was almost always the fastest according to the information given to participants. Contrary, in the OD pair with the lowest minimisation rate, either the alternative with high composition of freeway was not the fastest, or the three alternative routes were similar in their attributes. This can be observed in Appendix A and B; a formal analysis is presented in Section 4.3. The (weighted) mean of $F_{(1)}^{od}$ is equal to the global minimisation rate, i.e., $F_{(1)} = 0.605$, with a standard deviation of 0.16. The distribution of the minimisation rate at participant level is included in Fig. 8(b), where it can be seen that the travel time informed to participants does not have the same effect on all individuals. The group of perfect rational participants, who chose the fastest route in all of the choice problems, is relatively small, representing only 9.5% of the total number of participants. Moreover, the minimisation rate of participants is highly heterogeneous, showing a more or less even distribution between minimisation rates of 0.20 and 1.0. The mean of the minimisation rate by participant is 0.58, with a standard deviation of 0.24. This clearly highlights that the great majority of travellers do not make perfect rational decisions in all the choice problems they face, even when travel time estimates are available, thus suggesting a boundedly rational behaviour in route choice. As it will be shown later in this article, the variability in $F_{(1)}$ comes, primarily, from the travel time information in the alternative routes, and secondly, from the route attributes that make a route more attractive to the travellers.

4.1.1. Perfect rationality and differences in the travel time information

Travellers are not necessarily travel time minimisers, but how does the difference in travel time information in competing routes influence the behaviour of travellers? Do travellers value absolute or relative differences in travel time? The distributions of the absolute differences in travel time information between the two fastest routes, ΔITT, and the relative differences $\%\Delta ITT$ are shown in Fig. 9.

The proportion of times that the kth fastest route was chosen in each route choice problem, $F_{(k)}$, is obtained for the subsets $S_1(\alpha)$, defined in Eq. (4), with α being the 20-quantiles of the distributions. Recall that the sets $S_1(\alpha)$ are those in which the difference between the fastest and second fastest route is at least α. The minimisation rate, $F_{(1)}$, and the proportions $F_{(2)}$ and $F_{(3)}$, estimated for the different subsets $S_1(\alpha)$, are shown in Fig. 10, where it can be noticed that, in general, the larger the difference in travel time information, the larger the minimisation rate for both $\Delta ITT_{1,2}$ and $\%\Delta ITT_{1,2}$. This result is not surprising; travel time is recognised as the most important variable in route choice. However, there are important differences between the minimisation rate when considering $\Delta ITT_{1,2}$ or $\%\Delta ITT_{1,2}$. The first difference is that in the case of $\Delta ITT_{1,2}$, the maximum minimisation rate is barely above 0.75, whereas for $\%\Delta ITT_{1,2}$ it can reach values little higher than 0.90. The second difference is that for $\Delta ITT_{1,2}$ the minimisation rate does not show an increasing trend, having a high decrease in $F_{(1)}$ for large values of $\Delta ITT_{1,2}$. This behaviour can be hardly explained, as one would expect that larger differences in travel time information would induce larger minimisation behaviour: why would I choose the fastest route when it is 2 min faster, but not when it is 8 min faster? This is not the case $\%\Delta ITT_{1,2}$, where $F_{(1)}$ shows an increasing trend.

To formalise the above findings, two logistic regressions are fitted to the data. In both regressions, the response variable Y is binary, taking the value $Y_i = 1$ if the participant chose the fastest route according to the given information; the regressors are $\Delta ITT_{1,2}$ or $\%\Delta ITT_{1,2}$. The results of the logistic regressions are presented in Table 3, and their predictions for $F_{(1)}$ for the subsets $S_1(\alpha)$ are plotted along the observed values of $F_{(1)}$ in Fig. 11. In both cases, the regressors are different from zero, with a significance level of 0.001, however, the Akaike information criterion

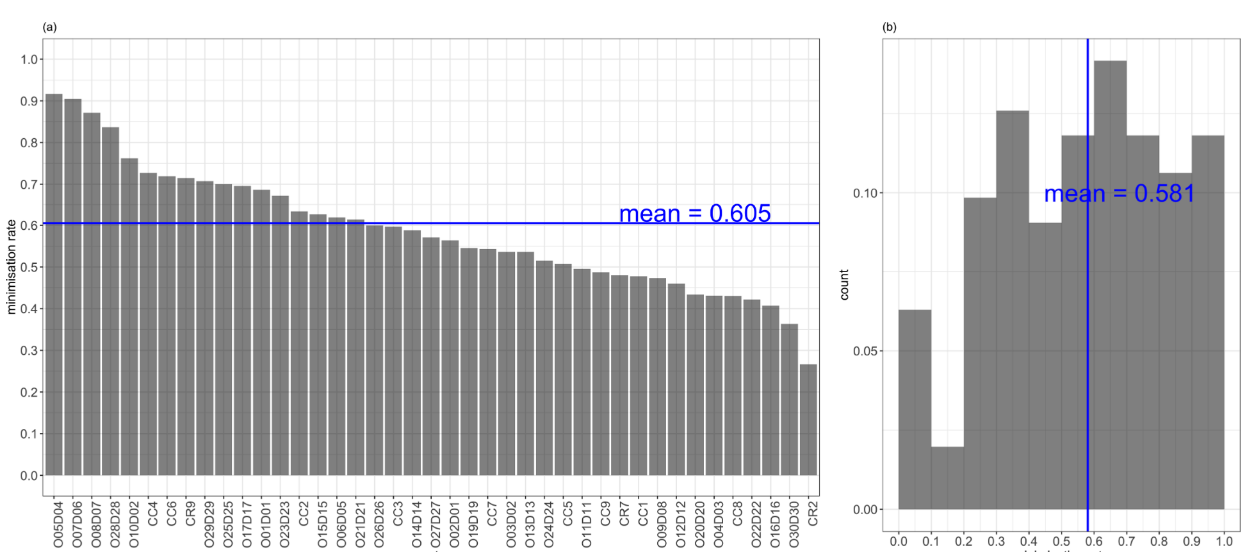

Fig. 8. (a) minimisation rates per OD pair, and (b) distribution of the minimisation rates of participants.

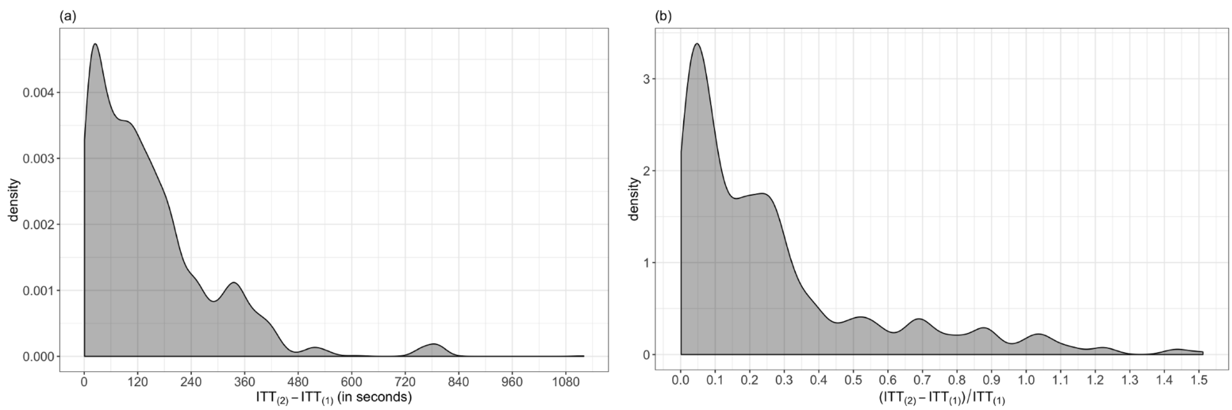

Fig. 9. Kernel density estimation of the distribution of (a) the absolute (seconds) and (b) the relative differences in travel time.

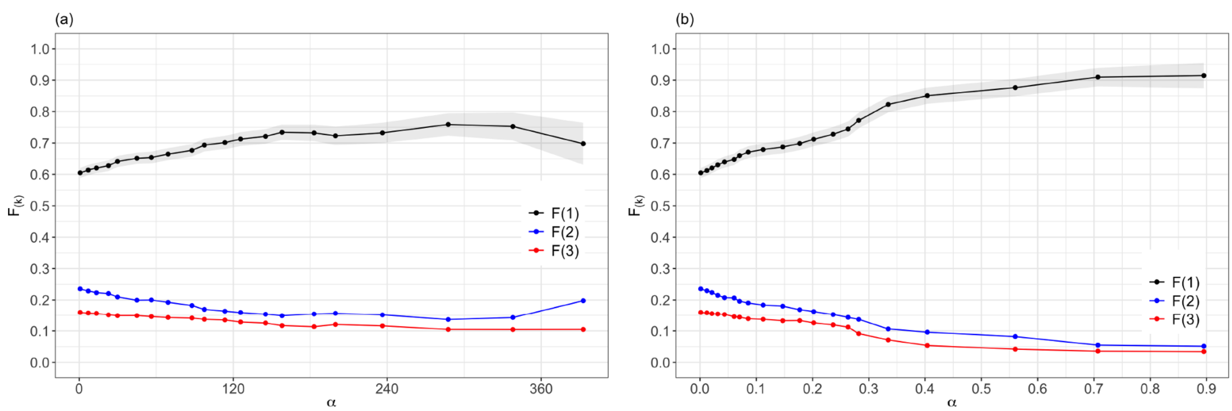

Fig. 10. Proportion of times the fastest, $F_{(1)}$, second fastest, $F_{(2)}$ and slowest, $F_{(3)}$ routes are chosen, computed for the subsets $S(p)$ for (a) the absolute difference in travel time information, and (b) the relative difference in travel time information. The ribbon corresponds to a margin of error of 5%.

(AIC) is smaller when using $\%\Delta ITT_{1,2}$ as a regressor, meaning a better fit with this explanatory variable. This result is confirmed by the Hosmer and Lemeshow goodness of fit test (Hosmer and Lemesbow, 1980). The null hypothesis H_0 of this test is that the observed proportions are similar to the predicted proportions in different subsets of the data. H_0 is rejected for $\Delta ITT_{1,2}$, but not for $\%\Delta ITT_{1,2}$, suggesting a *good* fit of the regression with $\%\Delta ITT_{1,2}$. Since the logistic regression is equivalent to the multinomial logit model when there are only two alternatives, this result can be interpreted from a behavioural perspective: $\%\Delta ITT_{1,2}$ is better than $\Delta ITT_{1,2}$ in explaining the minimisation behaviour of the participants, and therefore their route choice behaviour. This suggests that travellers evaluate the travel time in a problem-wise manner, i.e., relative to the travel time in the competing alternatives, and not as an absolute difference in time units.

4.2. Bounded rationality in route choice

In the previous section, the analysis was based on subsets $S_1(\alpha)$, i.e., the perfect rational behaviour. In this section, the bounded rationality of travellers is studied, so the cases when two or more routes are satisficing are analysed. First, the probability of choosing a satisficing route is estimated for different values of α for the case of choice problems with three alternative routes. Then, lower and upper bounds are derived for the general case when there are more than three alternative routes. In view of the above results, the analyses in this section are restricted to the relative differences $\%\Delta ITT$. As in the previous section, α is given by the 20-quantiles of the distribution of the travel time differences $\%\Delta ITT_{1,2}$.

The probability of choosing a satisficing route for the different values of α, $Pr(satisficing|\alpha)$, is shown in Fig. 12(a), along with the conditional probabilities $Pr(satisficing|S_n(\alpha))$, $n = 1, 2, 3$. The fraction of the data that each set $S_n(\alpha)$ represents, i.e., $Pr(S_n(\alpha))$, is presented in Fig. 12(b). In this last figure, it can be seen that the fraction of the choice problems in which there is only one satisficing route, $Pr(S_1(\alpha))$, decreases with α, while the fraction of problems with three satisficing routes, $Pr(S_3(\alpha))$, increases. This behaviour is expected, as larger indifference bands imply more satisficing alternatives. In Fig. 12(a), it can be observed that, in general, the probability of choosing a satisficing

Table 3
Summary of the logistic regressions with dependent variable $Y = 1$ when the fastest route was chosen, and regressors $\Delta ITT_{1,2}$ or $\%\Delta ITT_{1,2}$. The Hosmer and Lemeshow (H&L) goodness of fit test is included in the table.

Coefficient	Estimate (s.e.)	z statistic	$Pr(>\|z\|)$	Estimate (s.e.)	z statistic	$Pr(>\|z\|)$
intercept	−0.0305 (0.0517)	−0.59	0.555	−0.1556 (0.0484)	−3.218	0.0013
$\Delta ITT_{1,2}$	0.0033 (0.0003)	11.02	$<2e-16$			
$\%\Delta ITT_{1,2}$				2.4528 (0.1629)	15.058	$<2e-16$
H &L GOF	Deviance = 4,774.9 AIC = 4,778.9 $\chi^2 = 79.802$, df = 18, p-value = 9.282e−10			Deviance = 4,614.395 AIC = 4,618.4 $\chi^2 = 25.444$, df = 18, p-value = 0.1132		

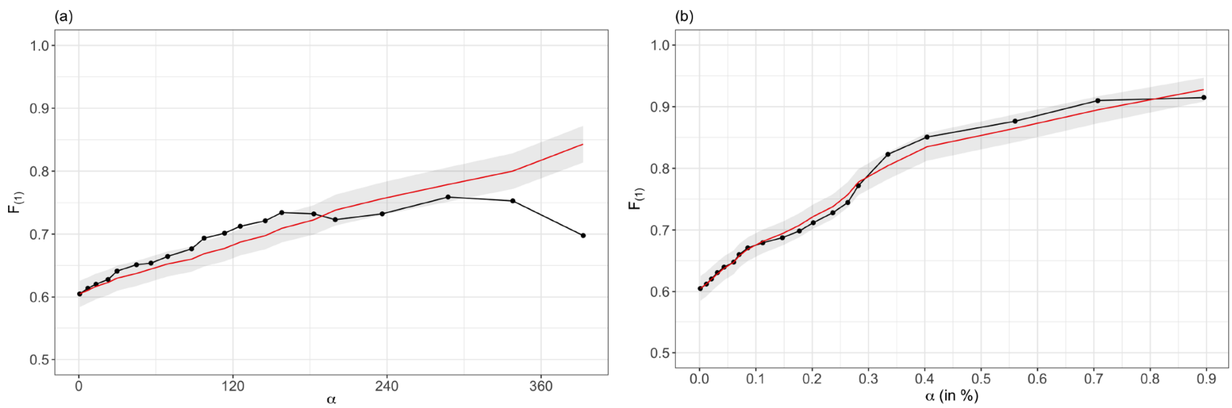

Fig. 11. Minimisation rate, $F_{(1)}$, computed for the subsets $S_1(\alpha)$ for (a) the absolute difference in travel time information, and (b) the relative difference in travel time information. The red lines are the predictions of the logistic models; the ribbon corresponds to the 95% confidence interval. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

route $Pr(satisficing|S_n(\alpha))$ increases with α, and that $Pr(satisficing|S_2(\alpha)) \geqslant Pr(satisficing|S_1(\alpha))$. These trends have a different cause. In the first case, the satisficing alternatives become more desirable as a consequence of a larger difference in the travel time information between the fastest route (which is always satisficing) and the not satisficing routes, i.e, larger values of α. In the second case, the trend is explained because, for the same value of α, the number of satisficing routes is larger in $S_2(\alpha)$ than in $S_1(\alpha)$, thus making it more likely to choose one. These two observations can be generalised to the case of choice problems with more than three alternative routes. In the first case, by arguing that the travel time has a negative effect on the choices of travellers. In the second case, by arguing that more satisficing alternatives imply necessarily less non-satisficing routes, so the probability of choosing a satisficing route is higher for larger values of *n*. Furthermore, more satisficing routes means a greater diversity of the route attributes, giving travellers more options from where to choose.

The probability $Pr(satisficing|\alpha)$ shown in Fig. 12(a), was estimated for choice problems with three alternative routes, but how would it look in the general case, i.e., for more than three alternatives? To answer this question, first note that since there are only three alternative routes in the choice problems, the probability $Pr(satisficing|S_3(\alpha)) = 1$ for all α. Assuming that the probabilities $Pr(satisficing|S_n(\alpha))$ and their weights $Pr(S_n(\alpha))$ ($n = 1$ and $n = 2$) in the general case can be estimated from the case of three alternatives, then the total probability estimated here, $\widehat{Pr}(satisficing|\alpha)$, overestimates the *real* total probability that would be observed in the presence of more than three alternative routes. Therefore, $\widehat{Pr}(satisficing|\alpha)$ can be considered as an upper bound for this *real* probability. To see this,

$$\begin{aligned} Pr\left(satisficing\middle|\alpha\right) &= \sum_{n=1}^{N} Pr\left(satisficing\middle|S_n(\alpha)\right) \times Pr\left(S_n(\alpha)\right) \\ &\leqslant \sum_{n=1}^{2} Pr\left(satisficing\middle|S_n(\alpha)\right) \times Pr\left(S_n(\alpha)\right) + \sum_{n=3}^{N} Pr\left(S_n(\alpha)\right) \\ &= \sum_{n=1}^{2} Pr\left(satisficing\middle|S_n(\alpha)\right) \times Pr\left(S_n(\alpha)\right) \\ &\quad + \left(1 - \sum_{n=1}^{2} Pr\left(S_n(\alpha)\right)\right) \\ &\approx \sum_{n=1}^{2} \widehat{Pr}\left(satisficing\middle|S_n(\alpha)\right) \times \widehat{Pr}\left(S_n(\alpha)\right) + \widehat{Pr}\left(S_3(\alpha)\right), \end{aligned} \tag{12}$$

where the inequality is obtained by making $Pr(satisficing|S_n(\alpha)) = 1$ for all $n \geqslant 3$, and the equality since $\sum_{n=1}^{N} Pr(S_n(\alpha)) = 1$ (they are disjoint events). To obtain a lower bound, recall from the previous analysis that it can be assumed that $Pr(satisficing|S_{n+1}(\alpha)) \geqslant Pr(satisficing|S_n(\alpha))$ for all *n*. Thus,

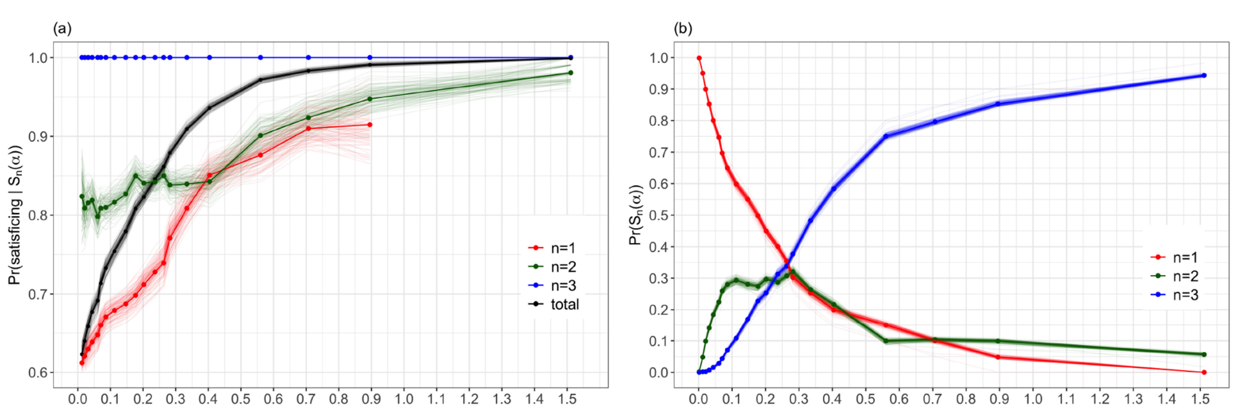

Fig. 12. (a) Conditional, $Pr(satisficing|S_n(\alpha))$, and unconditional, $Pr(satisficing\,|\,\alpha)$, probability of choosing a satisficing route as a function of α. (b) Probability of observing $S_n(\alpha)$ in the data, i.e, the fraction of observations with $n = 1, 2, 3$ satisficing routes for different values of α. These probabilities, computed for the bootstrap subsamples, are also included in the figures with lighter colours.

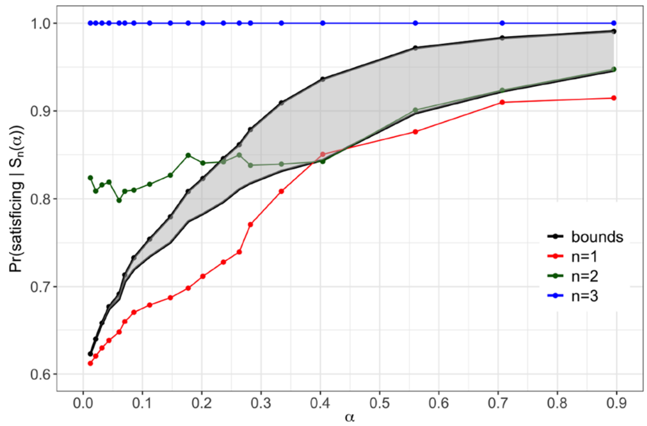

Fig. 13. Total probability of choosing a satisficing route for the general case (grey area). Perfect rationality is represented by the red line. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

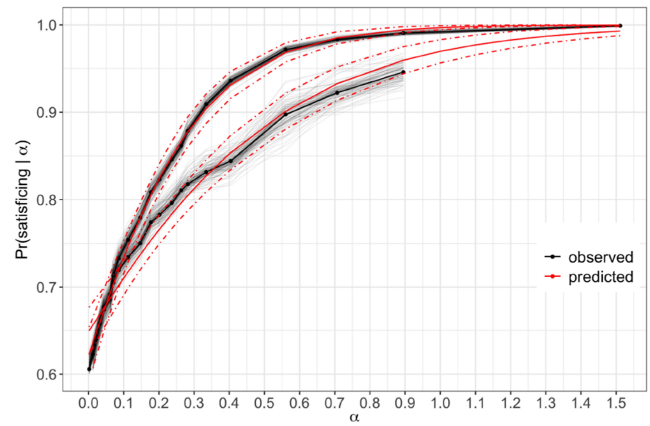

Fig. 14. Predicted upper and lower bounds for the probability of choosing a satisficing route. The 95% prediction error interval is represented with a dashed line.

$$
\begin{aligned}
Pr\left(satisficing\middle|\alpha\right) &= \sum_{n=1}^{N} Pr\left(satisficing\middle|S_n(\alpha)\right) \times Pr\left(S_n(\alpha)\right)\\
&\geqslant \sum_{n=1}^{2} Pr\left(satisficing\middle|S_n(\alpha)\right) \times Pr\left(S_n(\alpha)\right)\\
&\quad + Pr\left(satisficing\middle|S_2(\alpha)\right) \sum_{n=3}^{N} Pr\left(S_n(\alpha)\right)\\
&\approx \sum_{n=1}^{2} \widehat{Pr}\left(satisficing\middle|S_n(\alpha)\right) \times \widehat{Pr}\left(S_n(\alpha)\right)\\
&\quad + \widehat{Pr}(satisficing|S_2(\alpha))\widehat{Pr}(S_3(\alpha)).
\end{aligned} \tag{13}
$$

The lower and upper bounds for the probability in the general case are shown in Fig. 13, along with the estimated conditional probabilities $\widehat{Pr}(satisficing|S_n(\alpha))$. The results show that the estimates of the proportion of boundedly rational choices are higher that the estimates for the perfect rational choices, and that the difference is higher for $\alpha < 0.35$. For $\alpha = 0.35$, the estimated proportion of rational choices is 82%, whereas for boundedly rational choices is between 84% and 92%.

In Fig. 12(a), it can be seen that the estimates for the bootstrap subsamples do not differ considerably from the estimate considering all data, specially for the unconditional probability $Pr(satisficing|\alpha)$. This implies that, at aggregated level, the heterogeneity of participants and route attributes have little impact on the probability of choosing a satisficing route. As in the previous section, a logistic regression is fitted to the data to obtain a mathematical expression for the upper and lower bounds in the general case. The regression is fitted to the bootstrap subsamples to produce some variation. The results of the models are summarised in Table 4, where it can be seen that, for both cases, the regressor α is statistically significant (significance level 0.001), and that the Hosmer and Lemeshow goodness of fit test do not reject the null hypothesis that the observed and predicted probabilities are the same. Therefore, the upper and lower bounds for the probability of choosing a satisficing route, given the size of the indifference band α can be approximated by the logistic functions

$$
Pr(satisficing\,|\,\alpha)^{upper} = \frac{e^{0.49+5.23\alpha}}{1+e^{0.49+5.23\alpha}}
$$
$$
Pr(satisficing\,|\,\alpha)^{lower} = \frac{e^{0.61+2.85\alpha}}{1+e^{0.61+2.85\alpha}}.
$$

These bounds are shown in Fig. 14 along with the observed values of the bootstrap subsamples.

The conditional probabilities $Pr(satisficing|S_n(\alpha))$ can be decomposed as the sum of the more simple probabilities $Pr(C = I_{(k)}\,|\,S_n(\alpha))$, i.e., the sum of the probabilities of choosing the kth fastest route, given that there are n satisficing routes (see Eq. (5)). This decomposition is shown in Fig. 15, where it can be seen that the probability of choosing the fastest route is higher, no matter the value of α, i.e., $\widehat{Pr}(C = I_{(1)}\,|\,S_n(\alpha)) > \widehat{Pr}(C = I_{(2)}\,|\,S_n(\alpha))$ for all values of α. As expected, the preference for the fastest route amongst the satisficing routes increases with increasing values of α, however, it is interesting to note that the preference for the fastest route is much higher even for small values of α. This means that informing a route to be the fastest has already an effect on the preferences of the participants, regardless of the difference in the travel time with the rest of the alternatives. This effect is specially important in the case of the perfect rational travellers (9.5% in this study), who will always choose the fastest route. The probability of choosing the fastest route is approximately 29% higher in the case of $S_2(\alpha)$ and 116% higher in the case $S_3(\alpha)$.

4.2.1. Heterogeneity of the indifference band

Participants are heterogeneous in their indifference bands. This can be observed in Fig. 13 of the previous section, where $\widehat{Pr}(satisficing|\alpha) < 1$, contradicting the boundedly rational hypothesis

Table 4
Summary of the logistic regressions to approximate the upper and lower bands for $Pr(satisficing|S_n(\alpha))$ in the general case. The Hosmer and Lemeshow (H&L) goodness of fit test is included in the table.

	Upper bound			Lower bound		
Coefficient	Estimate (s.e.)	z statistic	$Pr(>\lvert z\rvert)$	Estimate (s.e.)	z statistic	$Pr(>\lvert z\rvert)$
intercept	0.4943 (0.0676)	7.313	2.62e−13	0.6122 (0.0620)	9.872	<2e−16
α	5.2326 (0.3962)	13.206	< 2e−16	2.8531 (0.2690)	10.606	<2e−16
H &L GOF	Deviance = 2.0312 AIC = 1367.2 χ^2 = 1.1096, df = 8, p-value = 0.9975			Deviance = 6.4038 AIC = 1578.5 χ^2 = 4.6106, df = 8, p-value = 0.7983		

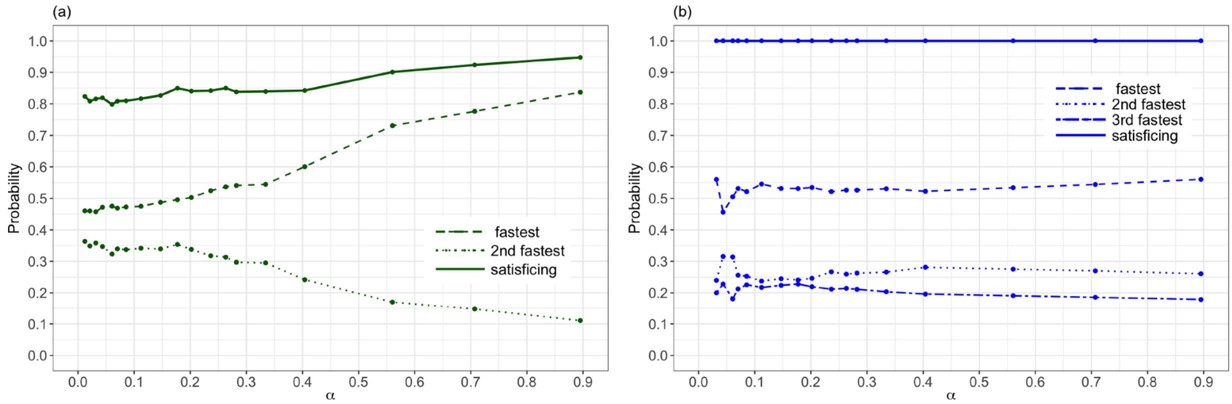

Fig. 15. Probabilities of choosing the kth fastest route amongst the n satisficing routes for (a) $n = 2$ and (b) $n = 3$.

that travellers choose satisficing routes. To put it another way, if participants were all boundedly rational with the same indifference band given by α^*, then $\widehat{Pr}(satisficing|\alpha) = 1$ for all the values of $\alpha \geqslant \alpha^*$. This is clearly not the case, unless a large (and therefore not meaningful) value of α^* is considered. In this section, the heterogeneity of the indifference bands is analysed. To this purpose, the estimators for the indifference band at individual level, α_i, defined in Section 3.2.1, are computed.

The distribution of the estimators $\hat{\alpha}_i^{max}$, $\hat{\alpha}_i^{95}$ and $\hat{\alpha}_i^{50}$ are presented in Fig. 16. It can be observed that for the estimators $\hat{\alpha}_i^{max}$ and $\hat{\alpha}_i^{95}$ the proportion of perfectly rational participants, as it was found in Section 4.1, is 9.5% ($\hat{\alpha}_i = 0$). Furthermore, there is a group of participants with large indifference band, $\alpha_i > 1$, meaning that they will still consider routes two times slower than the fastest route. The percentage of participants with these large indifference bands is 5% and 2% for $\hat{\alpha}_i^{max}$ and $\hat{\alpha}_i^{95}$, respectively. It is likely that these participants were not engaged in the experiments, as it is difficult to believe that a traveller is willing to choose a route twice as slow as the fastest alternative. For these two estimators, a large heterogeneity is observed, with values more or less uniformly distributed in the interval (0.15, 0.5]; in both cases, around half of the observations lie in this interval. However, the distribution of $\hat{\alpha}_i^{95}$ accumulates around 30% of the observations in the interval [0, 0.15], whereas the distribution of $\hat{\alpha}_i^{95}$ accumulates around 20% in the same interval. This explains difference of 0.08 percentage points in the means of the distributions. In contrast, the distribution of the estimators $\hat{\alpha}_i^{50}$ tells a completely different story, showing low heterogeneity with around 80% of the observations having a very small indifference band: $\hat{\alpha}_i^{50} \leqslant 0.10$. Moreover, with this definition, the proportion of perfect rational participants would be 55%, which is high compared to the observed proportion of perfectly rational participants (9.5%). Considering that $\hat{\alpha}_i^{max}$ is a restrictive estimator, very sensitive to outliers, and that $\hat{\alpha}_i^{50}$ overestimates the perfect rationality, the estimator $\hat{\alpha}_i^{95}$ may be a good selection. This later estimator is conservative, in the sense that it will include the great majority of the routes choices within the indifference band, but being less sensitive to outliers.

To see the consequences of using the distinct estimators of α_i, the proportion of satisficing choices in the data set are computed assuming that the participants are heterogeneous and that their indifference bands are given by the three estimators. These proportions are presented in Table 5. As expected, when the estimators are defined as $\hat{\alpha}_i^{max}$, the probability of choosing a satisficing route is 100%, since $\hat{\alpha}_i^{max}$ was defined so all the observed choices are satisficing. By relaxing this

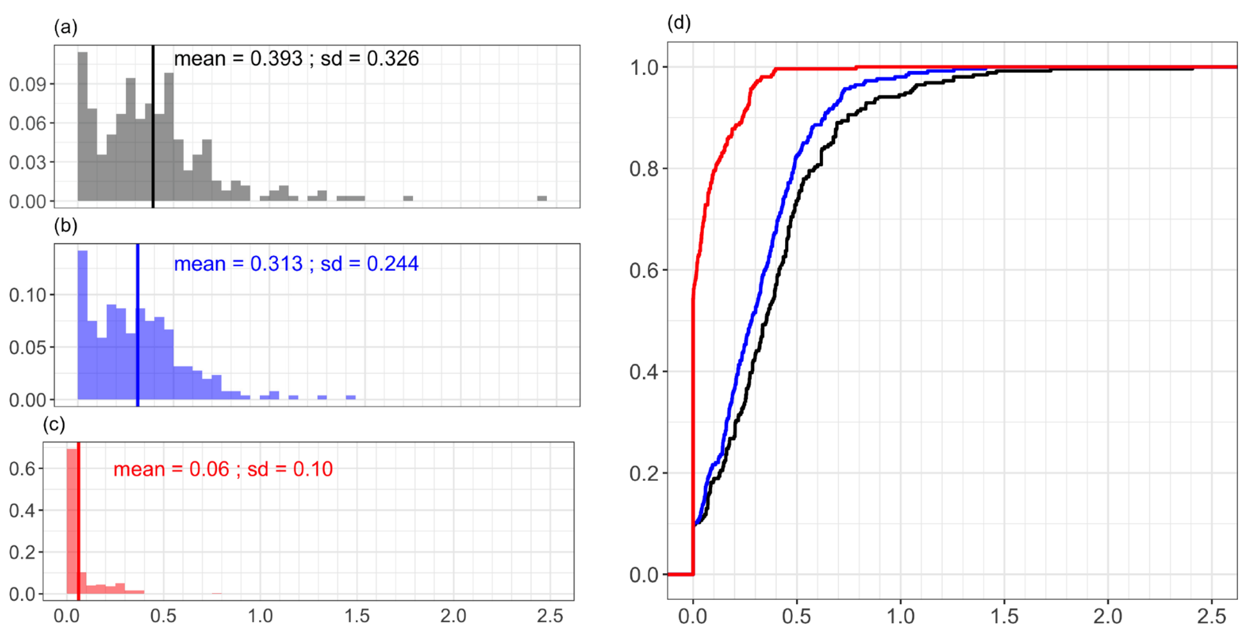

Fig. 16. Distributions of the indifference band by participant estimated using (a) the maximum of the travel time differences of the routes chosen by the participants, (b) the 95 percentile, and (c) the 50 median. The cumulative distributions are included in (d).

Table 5
Observed proportion of satisficing choices, (i) given different estimators for the individual indifference bands, α_i, and (ii) assuming homogeneity for the indifference band, $\hat{\alpha}_i$.

	$\hat{\alpha}_i^{\,max}$	$\hat{\alpha}_i^{\,95}$	$\hat{\alpha}_i^{\,50}$
Heterogeneous α_i	100%	89.9%	66.8%
Homogeneous $\alpha = \hat{\alpha}$	92.7%	88.9%	69.1 %

Table 6
Estimates for the mean and the covariance (standard deviation and correlation) of the parameters of the two MXL models *Model_0* and *Model_1* estimated for the participants that received travel time information. *lppd* is the log pointwise predictive density, an estimate of the predictive accuracy of the model: a higher value (compared to another model) means a better fit. WAIC (Watanabe-Akaike Information Criterion) penalises the *lppd* with the model complexity: a smaller value (compared to another model) means that the model represents a better alternative balancing goodness-of-fit and complexity. *err* is the discrepancy between the observed and predicted choice distributions. The significance of the estimated parameters is not tested.

	Model_0		*Model_1*	
Parameter of the posterior	Mean	s.error	Mean	s.error
$\hat{\bar{\beta}}_{FRW}$	0.862	0.313	0.622	0.340
$\hat{\bar{\beta}}_{DIR}$	1.377	0.640	1.863	0.753
$\hat{\bar{\beta}}_{TRN}$	0.012	0.108	0.061	0.132
$\hat{\bar{\beta}}_{INT}$	−0.044	0.029	−0.076	0.032
$\hat{\bar{\beta}}_{\%\Delta ITT}$	−3.285	0.356	0.366	0.373
$\hat{\sigma}^2_{FRW}$	2.840 (1.685)	2.253	3.596 (1.896)	1.842
$\hat{\sigma}^2_{DIR}$	17.889 (4.230)	11.229	25.970 (5.096)	9.383
$\hat{\sigma}^2_{TRN}$	0.505 (0.711)	0.185	0.464 (0.681)	0.178
$\hat{\sigma}^2_{INT}$	0.052 (0.228)	0.011	0.058 (0.240)	0.014
$\hat{\sigma}^2_{\%\Delta ITT}$	17.121 (4.138)	3.230	10.867 (3.297)	2.575
$\hat{\sigma}_{FRW,DIR}$	4.660 (0.654)	4.628	7.470 (0.773)	3.564
$\hat{\sigma}_{FRW,TRN}$	0.389 (0.324)	0.478	0.543 (0.420)	0.451
$\hat{\sigma}_{FRW,INT}$	0.011 (0.028)	0.092	−0.031 (-0.069)	0.093
$\hat{\sigma}_{FRW,\%\Delta ITT}$	0.116 (0.017)	1.843	0.882 (0.141)	1.596
$\hat{\sigma}_{DIR,TRN}$	1.175 (0.391)	1.025	1.622 (0.467)	1.057
$\hat{\sigma}_{DIR,INT}$	−0.340 (−0.352)	0.227	−0.486 (−0.397)	0.274
$\hat{\sigma}_{DIR,\%\Delta ITT}$	9.290 (0.531)	3.853	8.525 (0.507)	3.696
$\hat{\sigma}_{TRN,INT}$	−0.029 (-0.180)	0.032	−0.025 (−0.151)	0.035
$\hat{\sigma}_{TRN,\%\Delta ITT}$	−0.530 (−0.180)	0.530	0.021 (0.009)	0.543
$\hat{\sigma}_{INT,\%\Delta ITT}$	−0.336 (−0.356)	0.160	−0.382 (−0.482)	0.150
lppd	−3307.05		−3334.77	
WAIC $= -2lppd + 2p_{waic}$	6,632.27 (p_{waic} = 9.08)		6,688.20 (p_{waic} = 9.33)	
err	0.110%		0.104	

condition, and defining the estimator as $\hat{\alpha}_i^{\,95}$, the observed probability of choosing a satisficing route is 89.9%, and 66.8% for the estimator $\hat{\alpha}_i^{\,50}$. These proportions are similar to those obtained by assuming that participants are homogeneous with α equal to the means of the distributions. Note that the homogeneous case is equivalent to evaluating $\hat{\alpha}_i$ in Fig. 13. For $\hat{\alpha}_i^{\,max}$ and $\hat{\alpha}_i^{\,95}$, this implies that by assuming homogeneity and an indifference band equal to the mean, it is possible to know with high probability (92.7 and 88.9%) which routes are travel time satisficing. However, a smaller indifference band is preferred, as it may reduce the number of alternative routes that are considered.

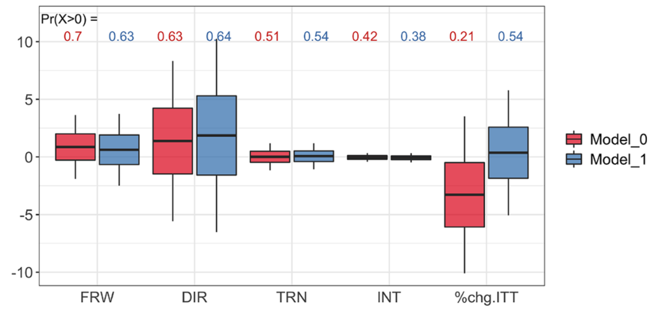

Fig. 17. Distributions of the random coefficients $\beta \sim \mathcal{N}(\hat{\bar{\beta}}, \hat{\Sigma})$ for models *Model_0* and *Model_1*. The probability of a coefficient being greater than zero in shown at the top of the plot.

4.3. Estimating the MXL model for route conditioned on the indifference band

In this section, the route choice probability is obtained by estimating the route choice model presented in Section 3.3. Name this model *Model_1*. The results are presented alongside the estimates of a second unrestricted MXL model (*Model_0*) that considers no indifference band. That is, in *Model_0* the three alternative routes are always considered by the decision maker and have a probability of being chosen greater than zero. The purpose of including *Model_0* in the analysis is to investigate how considering the indifference bands change user route choice behaviour. The models are compared in terms of their goodness-of-fit and predictive accuracy for out-of-sample observations at the end of this section. Both models were estimated using only the participants that received travel time information (353 participants and 3664 choices). It is worth mentioning that, in order to facilitate the interpretation and comparison between the two models, the informed travel time variable is considered in the specification of *Model_1* for the alternatives inside the indifference band. The Gibbs sampler software JAGS (Plummer, 2003) and the R (R Core Team, 2018) package *rjags* were used to obtain samples from the posterior distribution of the parameters β_i, $\bar{\beta}$ and Σ. The values of the hyperparameters, which define the priors of $\bar{\beta}$ and Σ, were chosen to be weakly-informative (very high variances). In other words, it is assumed high uncertainty on the real values of the parameters that are being estimated. The estimates for models *Model_0* and *Model_1* are presented in Table 6; the complete summary of the estimates is included in C.

Comparing the two models, it can be seen that there is a large difference in the distribution of the coefficients $\beta_{\%\Delta ITT}$, and that this difference is explained by a change in their mean values $\hat{\bar{\beta}}_{\%\Delta ITT}$ rather than a change in their variance: *Model_1* exhibits a mean closer to zero. Note that the distributions of the rest of the attributes do not vary considerably (Fig. 17). *Model_0* has a negative mean preference for travel time information $\hat{\bar{\beta}}^{M0}_{\%\Delta ITT} < 0$, meaning that the average traveller finds longer travel times undesirable. At individual level ***i***, the preferences for *%ΔITT* show a high heterogeneity, as it can be deduced from the estimated standard deviation ($\hat{\sigma}_{\%\Delta ITT} = 4.138$). The proportion of participants with a negative preference for *%ΔITT* is $Pr(\beta^{M0}_{\%\Delta ITT} < 0) = 0.21$, i.e., four in five participants prefer shorter time routes. Moving on to *Model_1*. The estimates show a positive mean preference for the travel time information, $\hat{\bar{\beta}}^{M1}_{\%\Delta ITT} > 0$, result that may appear counter intuitive, as it is interpreted as the mean participant choosing longer routes. However, contrary to *Model_0* where 4/5 of participants show a preference for shorter routes, in *Model_1* $Pr(\beta^{M1}_{\%\Delta ITT} < 0) = 0.46$, i.e., there is no clear trend in the preferences for the travel time information. In words, it is equally likely to find an individual preferring shorter time routes than longer ones within the indifference band. This finding is in accordance with the bounded rational model assumption in this article: travellers are indifferent to travel time when choosing a route from the

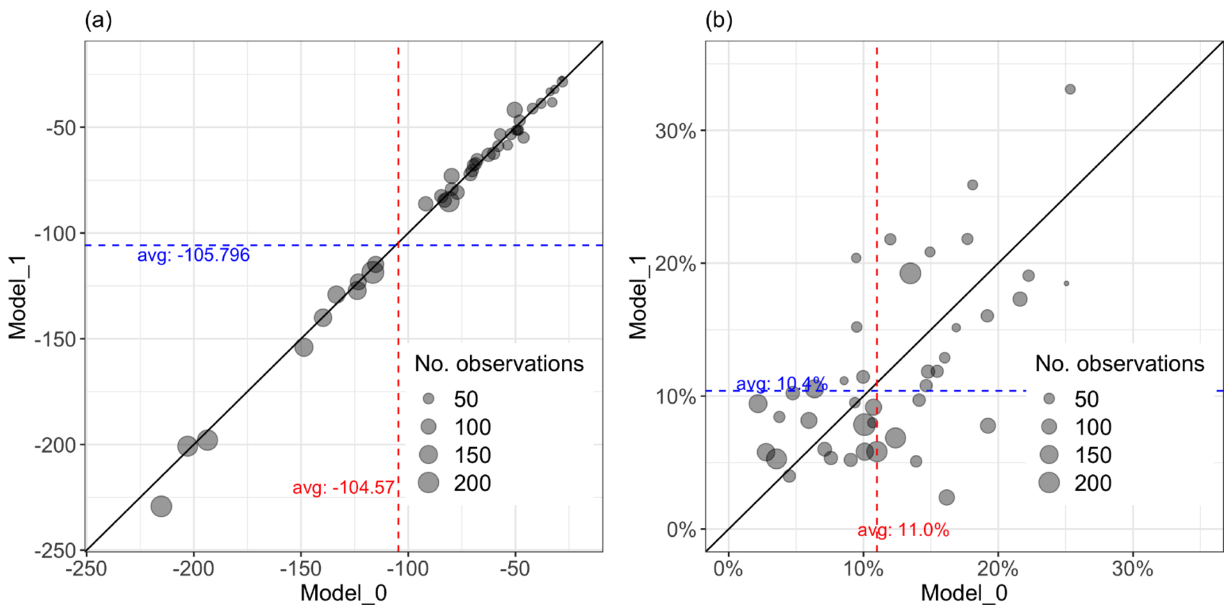

Fig. 18. Goodness-of-fit of models *Model_0* and *Model_1* evaluated by OD pair considering (a) the *lppd* and (b) *err* measures. The average is represented by the dashed lines, it is obtained considering the weight of each OD pair in the data set.

satisficing set. Furthermore, the physical attributes play, on average, a larger role in the choices of travellers in *Model_1* compare to *Model_0*. To see this, observe that there is no meaningful change in the distributions of the physical attributes between the models. Then, since the values of $\beta_{\%\Delta ITT}$ are closer to zero in *Model_1*, the importance of the travel time information relative to the rest of the attributes x, measured as $|\hat{\hat{\beta}}_{\%\Delta ITT}/\hat{\hat{\beta}}_x|$, decreases significantly. As in *Model_0*, the distribution of these coefficients show a high variance and, as a consequence, the coefficients $\beta_{\%\Delta ITT}$ may take large values. Nonetheless, the impact of this coefficient on the choice probabilities is lower than in the unrestricted model, as it multiplies smaller values of $\%\Delta ITT_{1,j}$ in the utility; routes with large differences would not be satisficing.

Continuing with the interpretation of the rest of the variables. A decrease in the mean preferences for *FRW* and *INT* can be noticed. In the case of *FRW*, the decrease places the average closer to zero, meaning that it becomes less important in the route choice. In the case of *INT*, the decrease makes it more important, meaning that within the indifference band a participant is less willing to choose a route with more intersections. It is important to mention that even though the coefficient seems small, this variable usually takes values $INT > 5$, making it an important variable defining the choice probabilities (see Fig. 3). This last variable is even more important than *FRW*. This is not the case for *TRN*, with coefficients near zero and taking values usually $TRN < 2$. The directness, *DIR*, is the most important attribute influencing the decisions of travellers in *Model_1*, but not in *Model_0* where $\%\Delta ITT$ dominates. It is interesting to note that *DIR* and $\%\Delta ITT$ are highly correlated (corr $= 0.5$) and that the correlation is positive. This implies that travellers who prefer direct routes are likely to prefer longer routes. This is true for both estimated models, suggesting that there may be two groups of travellers: one taking decisions mainly based on the travel time, and the other based on the directness or length of the trip.

Model_0 and *Model_1* are now evaluated in terms of their goodness-of-fit and their predictive accuracy. The goodness-of-fit is assessed by computing the *lppd* (expression (9)) and the discrepancy between the observed and predicted choice distributions *err* (expression (11)) using all the available observations. The estimation results in Table 6 show that the MXL model, *Model_0*, has a higher *lppd* than the boundedly rational model *Model_1* ($lppd^{M0} = -3307$ vs $lppd^{M1} = -3335$), meaning that, under *lppd*, the former model fits better the observations. The Watanabe-Akaike Information Criterion (WAIC), the Bayesian analogous of the Akaike Information Criterion (AIC) that penalises the goodness-of-fit by the complexity of the models, is smaller for *Model_0*. As it can be seen in Table 6, the factor that penalises for the complexity of the models, p_{waic}, is similar in both cases, meaning that the difference in WAIC between the two models is only explained by the *lppd*. This is not surprising, since both models estimate the same number of parameters: α_i^{max} in the case of *Model_1* enters as a variable, it is not a parameter estimated by the model. If model selection were based on the WAIC, then *Model_0* should be selected. However, in terms of the discrepancy between the observed and predicted choice distributions measured by the error *err*, the results are the opposite. The error *err* is computed OD pair wise and then averaged considering the weight of each OD pair in the observations. The results, respectively for *Model_0* and *Model_1* are 11.0% and 10.4%, meaning that in this case *Model_1* fits better the observed route choice distributions. For completeness of the results, both *lppd* and *err*, aggregated per OD pair and weighted by the number of observations in each OD pair are presented in Fig. 18, where it can be seen that models' performance is OD pair dependent and that no model is systematically superior to the other.

To complete this analysis, the predictive accuracy of the models is obtained for out-of-sample observations to assess how the models generalise to unobserved choices. For this purpose, bootstrapping is performed with 10 iterations. At each iteration, 1/3 of the observations are removed from the training set, the models are estimated with the training set and the *lppd* and *err* are computed for the out-of-sample observations. Bootstrap validation is used (instead of cross-validation) to leave out a sufficient number of observations that permit to compute the choice distributions of the 41 OD pairs. The results are summarised in Fig. 19. The results throw the same conclusions as in the above analysis: in terms of the *lppd*, *Model_0* performs slightly better in predicting new choices: the average *lppd* values across the ten iterations are $\bar{lppd}^{M0} = -1,101.65$ and $\bar{lppd}^{M1} = -1,116.49$, but in terms of *err* the results are the opposite with $err^{M0} = 12.65\%$ and $err^{M1} = 12.54\%$. The *lppd* is a measure related to the probability of observing the data, whereas *err* is a measure of discrepancy between the overall observed and predicted choice distributions. The opposite conclusions are explained because only the posterior predictive probability of the actual chosen alternatives contribute to the calculation of the *lppd*, while in

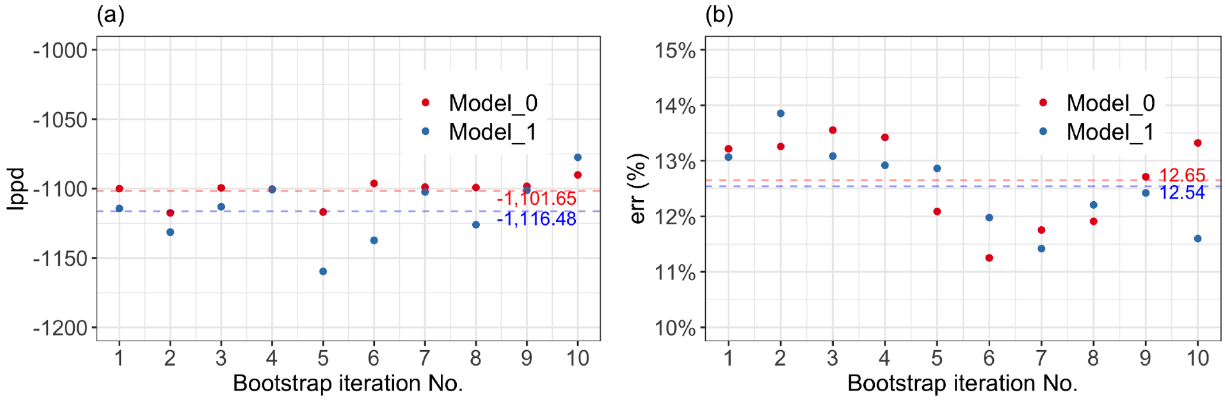

Fig. 19. Out-of-sample (a) log pointwise predictive density (*lppd*) and (b) discrepancy between the observed and predicted choice distributions (*err*) for each iteration of the bootstrap validation.

the *err* the posterior predictive probabilities of the forgone alternatives are also taken into account. The error *err* is interpreted as the percentage of trips that are erroneously assigned (on average) on a given OD pair. Thus, from a route choice point of view, *err* is more informative as it is related to the collective behaviour of travellers (distribution of choices over the OD pair alternative routes). This is of crucial importance in estimating the network loading. As a final conclusion, it can be said that both models have a similar predictive accuracy. However, *Model_1* is more in accordance than *Model_0* with the findings in the descriptive analysis in this article, where bounded rational behaviour is observed. Moreover, the difference in predictive accuracy between the models could be larger in favour on *Model_1* in cases with many alternatives per OD pair. In this case, some of the alternatives may be not satisficing for all individuals, and thus *Model_1* would assign a probability of being chosen equal to zero, unlike *Model_0* which assigns always positive probability to all alternatives.

5. Conclusions and discussion

In this work, the travel time minimisation behaviour and bounded rationality of travellers in route choice was studied through computer route choice experiments. In the experiments, participants made several route choices on 41 OD pairs presented over the road map of the city of Lyon. The choices of the participants were solely based on the travel time estimates (in minutes) and the map representation of the routes. It was found that, although participants received travel time estimates in the alternative routes, in 60.5% of the route choices participants chose the minimum travel time information route. This result lies within the range of those found in other studies (between 25% and 75%). However, it is important to take into consideration that the analysis presented here is based on route choices where participants received travel time information. Therefore, suggesting that in real-world situations, where travellers may not have travel time estimates on the forgone alternatives, the choices for the fastest route cannot be more than 60.5%. The percentage of choices for the fastest route was found to be OD pair and player dependant. According to the estimates of the MXL models, this dependency is explained by the heterogeneity of the preference of participants for the different route attributes, together with the variation of attributes between OD pairs. Apart from the travel time information, the directness of the routes resulted to be an important factor influencing the route choice of travellers.

The first main finding in this study is that travellers evaluate relative rather than absolute differences in travel time, at least for the ranges in travel time in the experiments. This means that a 5 min difference in travel time weights different for trips of 10 and 30 min. In the first case, the difference represents an increment of travel time of 50% with respect to the alternative, whilst in the second case the difference is of 15%. Therefore, the 5 min difference in the first case weighs more in favour of the fastest alternative. This implies that travellers minimise their travel time with respect to a reference point, given in this case by the travel time in the fastest route, and that the reference point is context-dependent, since it is evaluated in each route choice problem. This result has practical implications for the estimation of route choice models, and thus in traffic assignment, where expressing the travel time in relative terms could increase the realism of the predictions. For example, the travel times of the routes in each OD pair could be expressed as the percentage increase in travel time with respect to the minimum free flow travel time in that OD pair, or they can be transformed with the natural logarithm, as in the case of the Path Size Logit model (Ben-Akiva and Bierlaire, 1999), which also accounts for route overlapping. At individual level, a small percentage of the participants (10%) chose always the fastest route, these participants can be considered as perfect rational. The behaviour of the rest of the participants can be better explained by bounded rationality. In this regard, it was found that the participants are heterogeneous with respect to their indifference band, and that at least 70% of them would not consider routes with travel time differences 1.5 times slower than the fastest alternative. The mean indifference band can be estimated as 31.3%, meaning that the average participant did not consider routes with travel time differences 1.3 times slower than the fastest alternative. This value coincides with the average additional travel time in the choices observed in Hadjidimitriou et al. (2015). If travellers are assumed to be homogeneous with an indifference band equal to the mean, it is possible to know with high probability (88.9%) which routes are travel time satisficing. An interesting finding is that amongst the satisficing routes, the minimum travel time route was always preferred, even for small relative differences in travel time. This suggests that just the fact of informing a route to be the fastest increases its probability to be chosen. In this article the increase was of around 10 percentage points with respect to the second fastest route. A MXL model was estimated considering the heterogeneous indifference bands that define the satisficing alternatives for each participant (*Model_1*). This model was compared to the estimates of the classical MXL model that takes into account all the alternatives (*Model_0*). The results show that, as expected, travel time information losses explanatory importance in the first model, while the rest of the variables maintain their same level. Thus, amongst the satisficing alternatives, participants put more stress on the physical route attributes rather than on travel time information for their route choices. These models were compared in terms of their predictive accuracy for out-of-sample observations, resulting in similar predictive accuracy. When measured in terms of erroneously assigned trips for a given OD pair, the errors are around 12.6%. However, *Model_1* is more in accordance than *Model_0* with the bounded rational behaviour observed in the descriptive analysis in this article. This result is promising, considering that in *Model_1* the definition of the exogenous indifference band is α_i^{max} is restrictive. A bounded rational model that considers more flexible definitions for the indifference band could improve the performance. This would require to investigate more complex models capable of inferring the indifference bands endogenously. Moreover, *Model_1* could be more advantageous in choice situations with many alternatives, in which some alternatives will not be satisficing for all individuals and thus they will have probability equal to zero of being chosen. These questions are left as the subject of future investigation.

The findings in this article may have practical implications that are left for future work. In traffic simulation, the estimates for the indifference band can be used to reduce the search space of choice set generation by discarding routes with travel time differences (with respect to the shortest time route) above α; or used as exogenous inputs in bounded rational models as the one proposed in Watling et al. (2018). In these cases, the impact of considering homogeneous versus heterogeneous indifference bands could be assessed to determine the trade-offs between the simplicity of the former and the realism of the later. Apart from these two practical applications, the estimates of the indifference band can shed some light on the boundaries in which the users' route choices could be influenced, with the objective of directing them towards the social optimum (van Essen et al. (2016) provides a complete review on this subject).

CRediT authorship contribution statement

Humberto González Ramírez: Conceptualization, Methodology, Formal analysis, Investigation, Validation, Visualization, Writing - original draft, Writing - review & editing. **Ludovic Leclercq:** Methodology, Investigation, Writing - original draft, Writing - review & editing, Funding acquisition, Supervision, Project administration. **Nicolas Chiabaut:** Conceptualization, Methodology, Investigation, Writing - original draft, Writing - review & editing, Supervision. **Cécile Becarie:** Data curation, Resources, Software. **Jean Krug:** Data curation, Resources, Software.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Attributes of the OD pairs and routes

Definition of the variables:

- *EDIST*: the euclidean distance from origin to destination;*LEN*: the length of the route;
- *DIR*: the directness of the trip, defined as the euclidean distance divided by the length of the route;
- *TRN*: the number of turns per kilometre;
- *INT*: the number of intersections per kilometre;
- *FRW*: the % of freeway that composes the routes;
- *ITT*: travel time information (in minutes) that participants received.

Table A.7
Values of the attributes faced by player in the playable OD pairs for the Lyon-V36 network.

OD	Route	Map-reading						ITT			
		EDIST	*LEN*	*DIR*	*TRN*	*INT*	*FRW*	min	max	mean	s.d.
O01D01	R1	5.20	6.40	0.80	0.31	10.11	0.04	12.80	35.90	18.50	5.20
	R2	5.20	7.00	0.74	0.72	10.01	0.03	15.70	20.90	18.00	1.50
	R3	5.20	8.40	0.62	0.12	4.18	0.57	10.20	19.60	12.40	2.00
O02D01	R1	4.70	5.90	0.78	0.51	9.96	0.04	11.80	30.30	16.90	4.30
	R2	4.70	6.10	0.76	0.33	9.82	0.04	13.40	17.20	15.10	1.30
	R3	4.70	8.90	0.52	0.00	5.05	0.54	11.30	20.00	13.60	1.70
O03D03	R1	2.40	3.70	0.60	0.81	11.82	0.00	7.10	9.30	7.90	0.60
	R2	2.40	3.80	0.58	1.05	10.81	0.00	6.70	12.90	9.30	1.80
	R3	2.40	3.80	0.58	0.78	10.65	0.00	7.50	29.30	11.90	4.70
O04D03	R1	3.60	4.50	0.76	0.67	9.21	0.00	10.80	16.00	13.20	1.30
	R2	3.60	4.70	0.72	1.07	8.16	0.00	7.70	10.10	9.30	0.70
	R3	3.60	5.10	0.66	0.39	5.29	0.31	9.00	11.30	10.10	0.70
O05D04	R1	4.40	5.90	0.54	0.85	8.62	0.03	11.60	23.20	16.40	3.20
	R2	4.40	6.30	0.51	0.00	3.18	0.75	5.10	6.10	5.60	0.30
	R3	4.40	6.60	0.49	0.61	9.84	0.03	10.40	15.10	11.80	1.30
O06D05	R1	5.20	5.90	0.86	0.17	9.22	0.04	12.30	15.40	13.80	0.80
	R2	5.20	6.80	0.74	0.88	10.13	0.04	14.60	35.30	20.40	5.00
	R3	5.20	9.60	0.53	0.21	5.43	0.50	15.50	24.30	17.80	2.10

(continued on next page)

Table A.7 (*continued*)

OD	Route	Map-reading						ITT			
		EDIST	*LEN*	*DIR*	*TRN*	*INT*	*FRW*	min	max	mean	s.d.
O07D06	R1	3.50	5.50	0.60	0.91	8.60	0.03	12.40	31.40	17.30	4.50
	R2	3.50	6.40	0.52	0.79	3.62	0.77	6.00	6.60	6.30	0.20
	R3	3.50	6.40	0.51	1.25	9.19	0.03	9.30	11.40	10.00	0.60
O08D07	R1	2.90	3.30	0.69	0.60	3.59	0.28	3.40	4.00	3.50	0.10
	R2	2.90	3.40	0.68	1.17	10.27	0.00	5.80	6.70	6.20	0.30
	R3	2.90	4.00	0.59	1.52	8.59	0.00	6.30	7.10	6.60	0.20
O09D08	R1	3.10	4.00	0.78	0.75	8.48	0.05	7.50	16.90	9.40	2.20
	R2	3.10	4.30	0.73	0.70	8.38	0.05	9.50	12.50	10.80	0.90
	R3	3.10	5.10	0.62	0.20	9.41	0.38	10.00	11.40	10.90	0.40
O10D02	R1	3.10	4.90	0.59	2.02	8.49	0.04	7.40	9.40	8.00	0.60
	R2	3.10	5.20	0.56	0.58	8.14	0.04	9.80	14.40	11.70	1.20
	R3	3.10	5.40	0.54	0.93	8.96	0.04	9.30	11.10	10.20	0.50
O11D11	R1	2.70	3.80	0.66	1.84	11.30	0.00	8.20	11.10	9.50	0.90
	R2	2.70	4.00	0.63	2.01	9.80	0.00	9.10	11.80	10.20	0.80
	R3	2.70	4.70	0.53	0.85	8.07	0.00	8.20	9.00	8.60	0.10
O12D12	R1	2.70	3.90	0.59	0.77	7.48	0.00	7.80	9.10	8.40	0.30
	R2	2.70	4.20	0.54	1.91	9.56	0.00	8.00	9.40	8.50	0.40
	R3	2.70	4.60	0.50	1.09	9.81	0.00	8.50	10.60	9.50	0.50
O13D13	R1	3.20	3.90	0.74	0.78	9.84	0.00	9.70	12.70	11.40	0.80
	R2	3.20	4.30	0.66	2.09	11.62	0.00	8.60	13.50	9.60	0.80
	R3	3.20	5.00	0.57	1.41	9.23	0.00	11.00	13.60	12.40	0.70
O14D14	R1	2.80	3.40	0.62	2.36	10.02	0.00	6.30	9.80	7.70	1.30
	R2	2.80	3.80	0.55	1.57	9.92	0.00	6.30	11.70	8.00	1.20
	R3	2.80	4.00	0.52	0.99	10.88	0.00	7.90	10.80	9.10	0.70
O15D15	R1	4.10	4.60	0.84	0.87	10.93	0.00	9.80	25.00	13.20	3.10
	R2	4.10	5.40	0.71	0.74	9.94	0.00	12.70	36.00	16.60	5.30
	R3	4.10	6.20	0.63	0.49	11.85	0.00	13.40	15.10	14.10	0.50

Appendix B. Maps of the OD pairs and routes

Table A.8
Values of the attributes faced by player in the playable OD pairs for the Lyon-full network.

OD	Route	Map-reading						ITT			
		EDIST	*LEN*	*DIR*	*TRN*	*INT*	*FRW*	min	max	mean	s.d.
O16D16	R1	10.70	13.70	0.78	0.15	5.57	0.79	16.30	18.10	16.40	0.30
	R2	10.70	18.90	0.56	0.16	3.02	0.78	16.20	18.30	17.20	0.70
	R3	10.70	21.00	0.51	0.14	2.58	0.73	22.30	27.70	23.80	1.40
O17D17	R1	10.50	14.80	0.70	0.41	8.58	0.27	27.60	32.80	28.90	1.00
	R2	10.50	15.20	0.68	0.00	3.55	0.75	15.80	17.60	16.50	0.40
	R3	10.50	19.30	0.54	0.26	3.42	0.81	18.30	21.00	19.50	0.80
O19D19	R1	6.40	7.90	0.78	0.66	9.79	0.00	19.70	24.00	21.60	1.30
	R2	6.40	8.50	0.71	1.21	10.19	0.00	20.80	24.70	22.40	1.00
	R3	6.40	10.80	0.57	1.14	9.84	0.27	25.10	31.20	27.40	1.60
O20D20	R1	6.10	9.00	0.67	0.57	12.18	0.25	17.00	21.40	18.10	1.00
	R2	6.10	10.10	0.60	0.61	9.90	0.02	23.30	27.30	25.00	1.00
	R3	6.10	15.70	0.38	0.19	4.64	0.65	17.80	19.10	18.60	0.30
O21D21	R1	4.10	5.00	0.76	1.05	10.20	0.00	12.90	15.70	14.40	0.90
	R2	4.10	5.40	0.70	0.97	11.42	0.25	13.80	17.90	16.10	1.20
	R3	4.10	6.40	0.60	0.81	10.94	0.00	12.90	17.00	14.60	1.10
O22D22	R1	8.20	10.20	0.80	0.40	9.60	0.18	19.70	31.90	22.00	3.20
	R2	8.20	11.20	0.73	0.73	9.12	0.00	27.30	43.60	30.40	3.00
	R3	8.20	17.60	0.46	0.46	4.70	0.45	22.60	29.70	25.60	2.20
O23D23	R1	6.40	8.40	0.78	1.10	9.29	0.66	13.30	18.20	14.60	1.20
	R2	6.40	10.10	0.65	1.32	9.52	0.03	19.80	22.20	20.50	0.70
	R3	6.40	10.50	0.62	1.07	7.68	0.00	23.70	26.30	24.00	0.50
O24D24	R1	9.20	14.20	0.62	1.29	8.95	0.00	32.10	42.60	34.20	2.20
	R2	9.20	14.60	0.60	0.56	8.49	0.24	27.80	39.60	31.70	3.10
	R3	9.20	17.10	0.52	0.53	5.97	0.37	27.30	51.40	33.60	6.50

(*continued on next page*)

Table A.8 (*continued*)

OD	Route	Map-reading						ITT			
		EDIST	*LEN*	*DIR*	*TRN*	*INT*	*FRW*	min	max	mean	s.d.
O25D25	R1	5.90	6.70	0.83	0.93	8.68	0.00	15.10	17.30	16.10	0.50
	R2	5.90	8.10	0.69	1.15	9.18	0.00	16.30	19.60	18.50	1.30
	R3	5.90	10.70	0.52	0.57	3.84	0.56	13.20	15.50	14.40	0.70
O26D26	R1	7.20	8.90	0.80	0.46	12.09	0.10	20.60	33.60	23.50	2.40
	R2	7.20	12.60	0.57	0.49	8.67	0.50	19.90	25.80	22.00	1.40
	R3	7.20	14.20	0.51	0.07	4.30	0.56	15.80	20.60	17.20	0.90
O27D27	R1	5.60	8.20	0.67	0.63	6.99	0.35	12.50	16.70	14.70	1.30
	R2	5.60	9.40	0.58	1.09	8.62	0.00	19.50	23.50	21.20	1.10
	R3	5.60	9.40	0.58	0.54	9.33	0.27	18.10	20.20	19.30	0.70
O28D28	R1	10.70	13.00	0.82	0.08	6.86	0.21	18.30	21.90	19.20	0.80
	R2	10.70	17.40	0.61	0.87	8.77	0.15	38.00	57.10	38.90	2.30
	R3	10.70	17.70	0.60	0.29	5.47	0.29	30.90	40.20	32.20	1.30
O29D29	R1	5.80	8.10	0.72	0.38	7.15	0.00	14.20	18.30	15.60	0.90
	R2	5.80	8.60	0.68	0.96	7.12	0.00	17.70	22.20	19.50	1.00
	R3	5.80	9.00	0.65	0.23	2.44	0.80	9.10	11.30	10.00	0.60
O30D30	R1	8.00	10.50	0.75	0.68	8.74	0.32	20.40	22.30	21.00	0.40
	R2	8.00	10.80	0.73	1.42	12.09	0.00	25.60	33.80	28.50	2.00
	R3	8.00	12.90	0.61	0.87	6.99	0.14	26.10	30.50	27.70	1.10
CC1	R1	6.30	8.40	0.70	0.86	9.42	0.25	15.20	22.00	18.00	2.50
	R2	6.30	8.70	0.68	1.19	11.21	0.09	17.10	21.40	19.00	1.20
	R3	6.30	10.30	0.57	0.79	8.03	0.35	19.90	22.70	21.40	1.00
CC2	R1	2.20	3.30	0.66	1.96	10.90	0.00	8.10	9.60	9.10	0.30
	R2	2.20	3.60	0.60	2.08	8.59	0.00	8.60	9.90	9.40	0.40
	R3	2.20	3.90	0.56	1.39	11.68	0.00	8.90	10.50	10.00	0.40
CC3	R1	6.80	9.90	0.66	0.72	10.67	0.29	19.10	34.10	26.30	4.30
	R2	6.80	10.30	0.64	0.70	9.02	0.14	23.00	31.40	26.10	2.10
	R3	6.80	11.10	0.59	0.65	9.28	0.13	22.80	28.10	24.80	1.30
CC4	R1	4.80	6.50	0.71	1.13	10.67	0.00	12.50	13.40	13.00	0.30
	R2	4.80	7.10	0.65	1.17	10.31	0.00	16.00	17.70	16.90	0.60
	R3	4.80	10.10	0.46	0.41	6.92	0.32	18.00	20.50	19.40	0.80
CC5	R1	6.60	9.90	0.67	0.73	11.33	0.07	24.10	34.60	28.40	3.20
	R2	6.60	12.30	0.54	0.66	9.03	0.35	22.90	32.80	25.80	2.30
	R3	6.60	15.70	0.42	0.77	5.14	0.50	22.50	26.70	24.40	1.20
CC6	R1	7.90	10.00	0.69	0.31	7.86	0.24	16.00	20.00	18.10	1.30
	R2	7.90	13.30	0.52	0.31	6.48	0.32	20.00	26.90	23.10	1.10
	R3	7.90	14.50	0.47	0.35	7.64	0.49	22.00	27.80	24.40	1.40
CC7	R1	3.80	5.40	0.68	1.16	11.43	0.15	14.50	17.80	15.90	0.70
	R2	3.80	8.00	0.46	0.78	5.91	0.33	10.20	21.90	14.20	3.50
	R3	3.80	9.50	0.39	0.65	6.97	0.52	13.20	16.20	14.40	0.90
CC8	R1	3.30	4.30	0.75	0.74	11.12	0.00	10.40	16.10	13.00	1.70
	R2	3.30	4.80	0.67	1.31	9.92	0.00	8.70	16.50	12.40	2.70
	R3	3.30	5.60	0.58	1.32	9.72	0.00	12.10	14.30	13.20	0.60
CC9	R1	3.00	5.20	0.55	1.43	11.06	0.00	11.60	14.30	12.50	0.80
	R2	3.00	5.80	0.49	0.90	8.95	0.12	12.10	14.90	13.30	0.80
	R3	3.00	6.90	0.42	1.06	9.18	0.29	11.80	15.10	12.90	0.70
CR2	R1	2.50	3.30	0.68	1.97	9.70	0.00	7.10	7.80	7.40	0.10
	R2	2.50	3.80	0.59	1.97	9.73	0.00	6.80	7.30	7.10	0.20
	R3	2.50	4.70	0.48	2.49	10.07	0.00	8.40	9.10	8.60	0.30
CR7	R1	4.80	8.80	0.52	1.52	10.09	0.00	21.10	23.10	21.90	0.60
	R2	4.80	9.40	0.49	0.65	8.07	0.00	20.40	22.70	21.20	0.70
	R3	4.80	15.90	0.29	0.77	4.65	0.49	21.50	26.90	22.60	1.40
CR9	R1	3.60	5.70	0.51	1.09	6.26	0.19	11.30	12.40	11.80	0.30
	R2	3.60	6.40	0.46	1.14	9.86	0.00	12.50	13.60	13.10	0.40
	R3	3.60	7.40	0.39	1.11	7.65	0.00	16.80	20.90	17.70	1.10

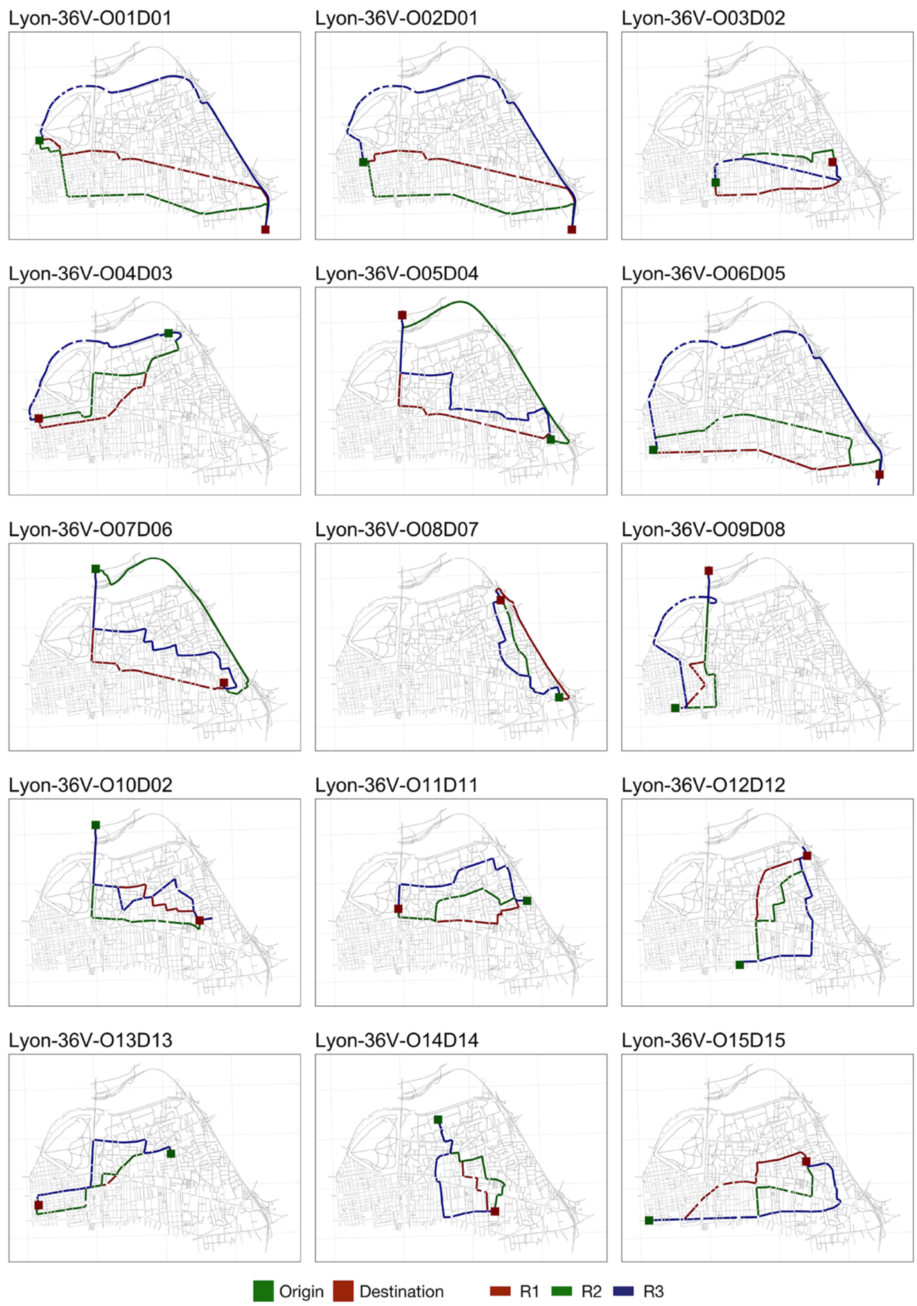

Fig. B.20. OD pairs and three connecting routes defined for the MDG experiments in the Lyon-V36 network.

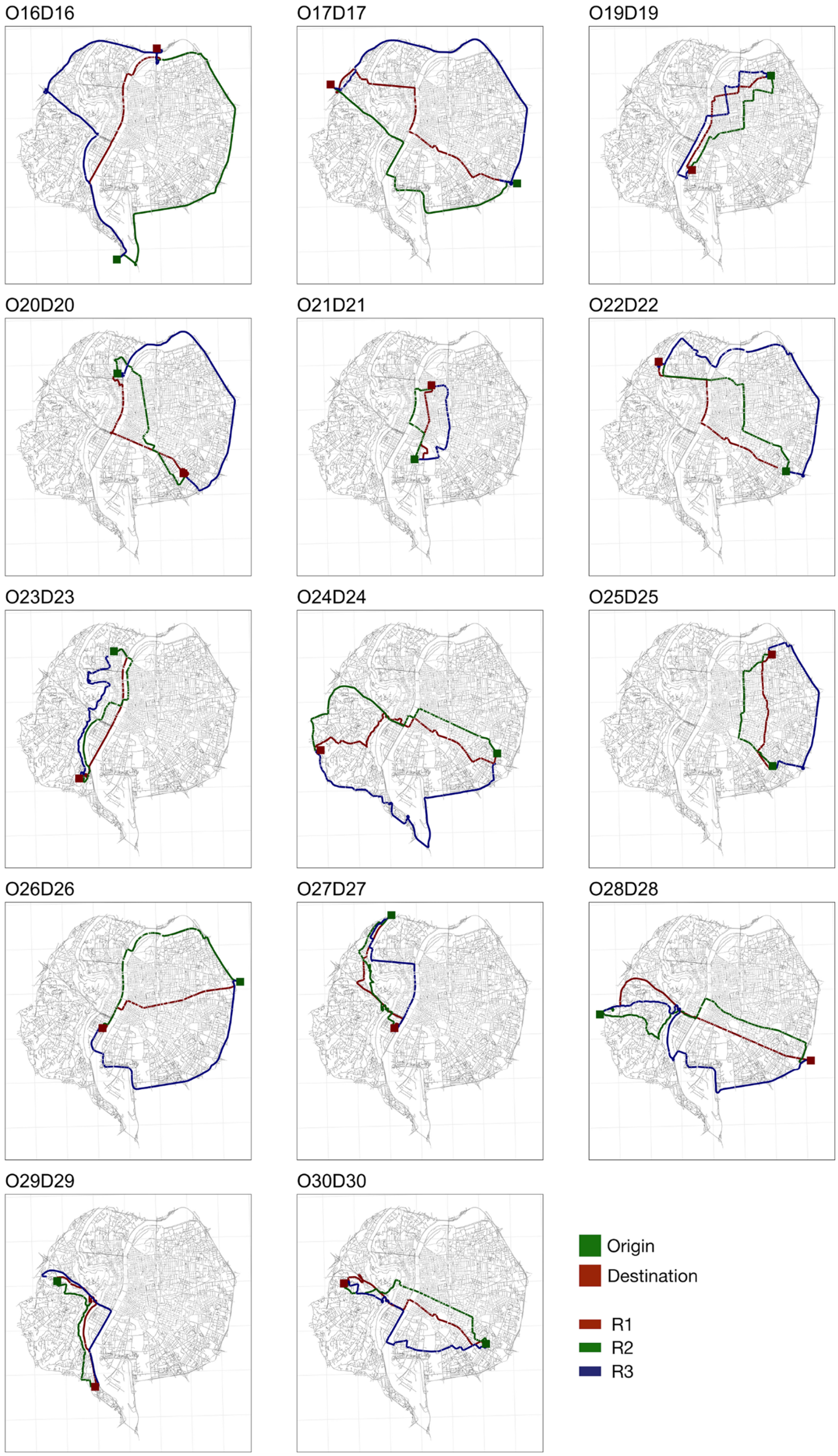

Fig. B.21. OD pairs and three connecting routes defined for the MDG experiments in the Lyon-full network.

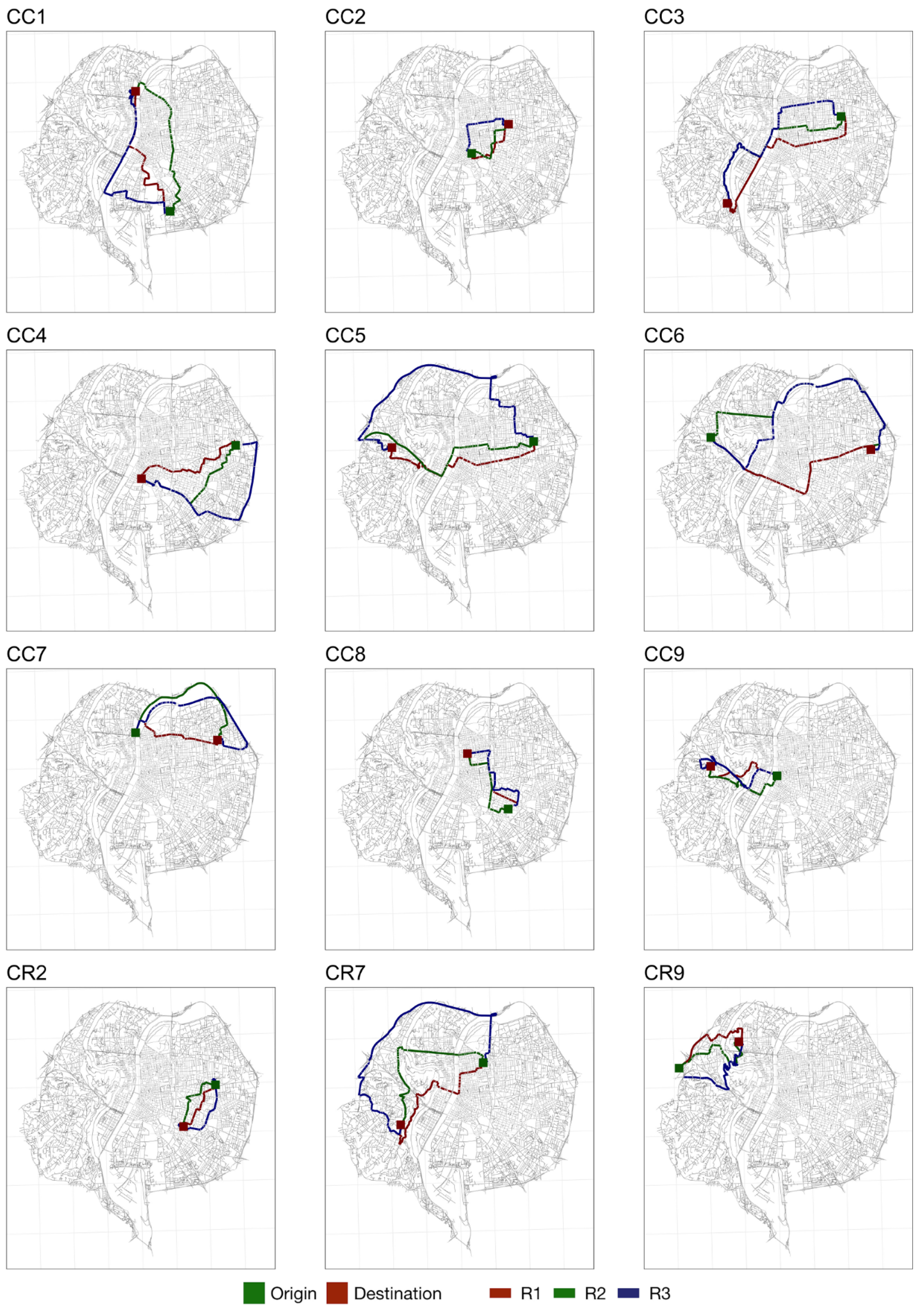

Fig. B.22. OD pairs and three connecting routes defined for the MDG experiments in the Lyon-full network.

Appendix C. Detailed results of the estimates for *Model_0* and *Model_1*

Table C.9
Results *Model_0*. Statistics on the posterior distribution for the mean and covariance matrix.

	Statistic*							
Parameter	Mean	s.d.	qt.2.5%	qt.25%	qt.50%	qt.75%	qt.97.5%	$\widehat{R}$**
$\widehat{\beta}_{FRW}$	0.862	0.313	0.242	0.658	0.869	1.065	1.485	1.026
$\widehat{\beta}_{DIR}$	1.377	0.640	0.018	0.969	1.414	1.838	2.502	1.042
$\widehat{\beta}_{TRN}$	0.012	0.108	−0.204	−0.059	0.014	0.086	0.218	1.021
$\widehat{\beta}_{INT}$	−0.044	0.029	−0.099	−0.063	−0.045	−0.024	0.011	1.007
$\widehat{\beta}_{\%\Delta ITT}$	−3.285	0.356	−4.010	−3.512	−3.276	−3.045	−2.604	1.018
$\widehat{\sigma}^2_{FRW}$	2.840	2.253	0.258	0.896	2.261	4.285	8.010	1.123
$\widehat{\sigma}^2_{DIR}$	17.889	11.229	2.304	9.003	16.272	24.554	44.478	1.165
$\widehat{\sigma}^2_{TRN}$	0.505	0.185	0.218	0.371	0.475	0.615	0.935	1.042
$\widehat{\sigma}^2_{INT}$	0.052	0.011	0.034	0.044	0.051	0.059	0.077	1.008
$\widehat{\sigma}^2_{\%\Delta ITT}$	17.121	3.230	11.608	14.842	16.833	19.134	24.192	1.001
$\widehat{\sigma}_{FRW,DIR}$	4.660	4.628	−1.565	0.599	3.936	7.758	14.893	1.122
$\widehat{\sigma}_{FRW,TRN}$	0.389	0.478	−0.379	0.039	0.298	0.714	1.460	1.091
$\widehat{\sigma}_{FRW,INT}$	0.011	0.092	−0.173	−0.045	0.015	0.069	0.191	1.022
$\widehat{\sigma}_{FRW,\%\Delta ITT}$	0.116	1.843	−3.147	−1.187	0.016	1.284	3.987	1.035
$\widehat{\sigma}_{DIR,TRN}$	1.175	1.025	−0.309	0.363	1.011	1.840	3.468	1.081
$\widehat{\sigma}_{DIR,INT}$	−0.340	0.227	−0.886	−0.465	−0.304	−0.178	0.014	1.004
$\widehat{\sigma}_{DIR,\%\Delta ITT}$	9.290	3.853	2.868	6.492	8.991	11.662	17.941	1.008
$\widehat{\sigma}_{TRN,INT}$	−0.029	0.032	−0.099	−0.048	−0.027	−0.007	0.028	1.007
$\widehat{\sigma}_{TRN,\%\Delta ITT}$	−0.530	0.530	−1.582	−0.880	−0.535	−0.172	0.479	1.001
$\widehat{\sigma}_{INT,\%\Delta ITT}$	−0.336	0.160	−0.675	−0.438	−0.328	−0.223	−0.042	1.005
$lppd = -3307.05$; WAIC***=6,632.27 (p_{waic} = 9.08)								

* Statistics based on 3,000 samples of the posterior after 80,000 (40,000 burn-in period) and saving 1/40 samples (thinning).
** R^2Potential Scale Reduction. When the MCMC chains converge, it takes values close to 1.
*** WAIC is an estimate of expected predictive error (lower WAIC is better).

Table C.10
Results *Model_1*. Statistics on the posterior distribution for the mean and covariance matrix.

	Statistic*							
Parameter	Mean	s.d.	qt.2.5%	qt.25%	qt.50%	qt.75%	qt.97.5%	$\widehat{R}$
$\widehat{\beta}_{FRW}$	0.622	0.340	−0.053	0.393	0.625	0.853	1.281	1.010
$\widehat{\beta}_{DIR}$	1.863	0.753	0.387	1.331	1.868	2.395	3.311	1.001
$\widehat{\beta}_{TRN}$	0.061	0.132	−0.197	−0.030	0.062	0.152	0.311	1.012
$\widehat{\beta}_{INT}$	−0.076	0.032	−0.139	−0.098	−0.076	−0.053	−0.014	1.006
$\widehat{\beta}_{\%\Delta ITT}$	0.366	0.373	−0.378	0.117	0.360	0.626	1.097	1.001
$\widehat{\sigma}^2_{FRW}$	3.596	1.842	0.669	2.247	3.381	4.671	7.766	1.039
$\widehat{\sigma}^2_{DIR}$	25.970	9.383	10.586	19.601	24.665	31.469	47.839	1.010
$\widehat{\sigma}^2_{TRN}$	0.464	0.178	0.188	0.329	0.440	0.570	0.861	1.005
$\widehat{\sigma}^2_{INT}$	0.058	0.014	0.035	0.048	0.056	0.066	0.089	1.001
$\widehat{\sigma}^2_{\%\Delta ITT}$	10.867	2.575	6.357	9.056	10.690	12.499	16.354	1.004
$\widehat{\sigma}_{FRW,DIR}$	7.470	3.564	1.634	4.831	7.176	9.757	15.380	1.039
$\widehat{\sigma}_{FRW,TRN}$	0.543	0.451	−0.224	0.237	0.490	0.813	1.525	1.028
$\widehat{\sigma}_{FRW,INT}$	−0.031	0.093	−0.219	−0.091	−0.033	0.031	0.154	1.019
$\widehat{\sigma}_{FRW,\%\Delta ITT}$	0.882	1.596	−2.146	−0.141	0.874	1.831	4.347	1.011
$\widehat{\sigma}_{DIR,TRN}$	1.622	1.057	−0.256	0.932	1.521	2.218	3.909	1.026

(*continued on next page*)

Table C.10 (*continued*)

Parameter	Statistic[*]							$\hat{R}$
	Mean	s.d.	qt.2.5%	qt.25%	qt.50%	qt.75%	qt.97.5%	
$\hat{\sigma}_{DIR,INT}$	−0.486	0.274	−1.075	−0.653	−0.465	−0.292	−0.017	1.002
$\hat{\sigma}_{DIR,\%\Delta ITT}$	8.525	3.696	1.888	5.925	8.295	10.930	15.974	1.003
$\hat{\sigma}_{TRN,INT}$	−0.025	0.035	−0.096	−0.047	−0.024	−0.002	0.043	1.009
$\hat{\sigma}_{TRN,\%\Delta ITT}$	0.021	0.543	−1.059	−0.333	0.034	0.393	1.069	1.024
$\hat{\sigma}_{INT,\%\Delta ITT}$	−0.382	0.150	−0.715	−0.477	−0.370	−0.276	−0.121	1.003
$lppd = -3334.77$; WAIC[***] = 6,688.20 ($p_{waic} = 9.33$)								

[**]R^2 Potential Scale Reduction. When the MCMC chains converge, it takes values close to 1.

[*] Statistics based on 3000 samples of the posterior after 80,000 (40,000 burn-in period) and saving 1/40 samples (thinning).

[***] WAIC is an estimate of expected predictive error (lower WAIC is better).

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